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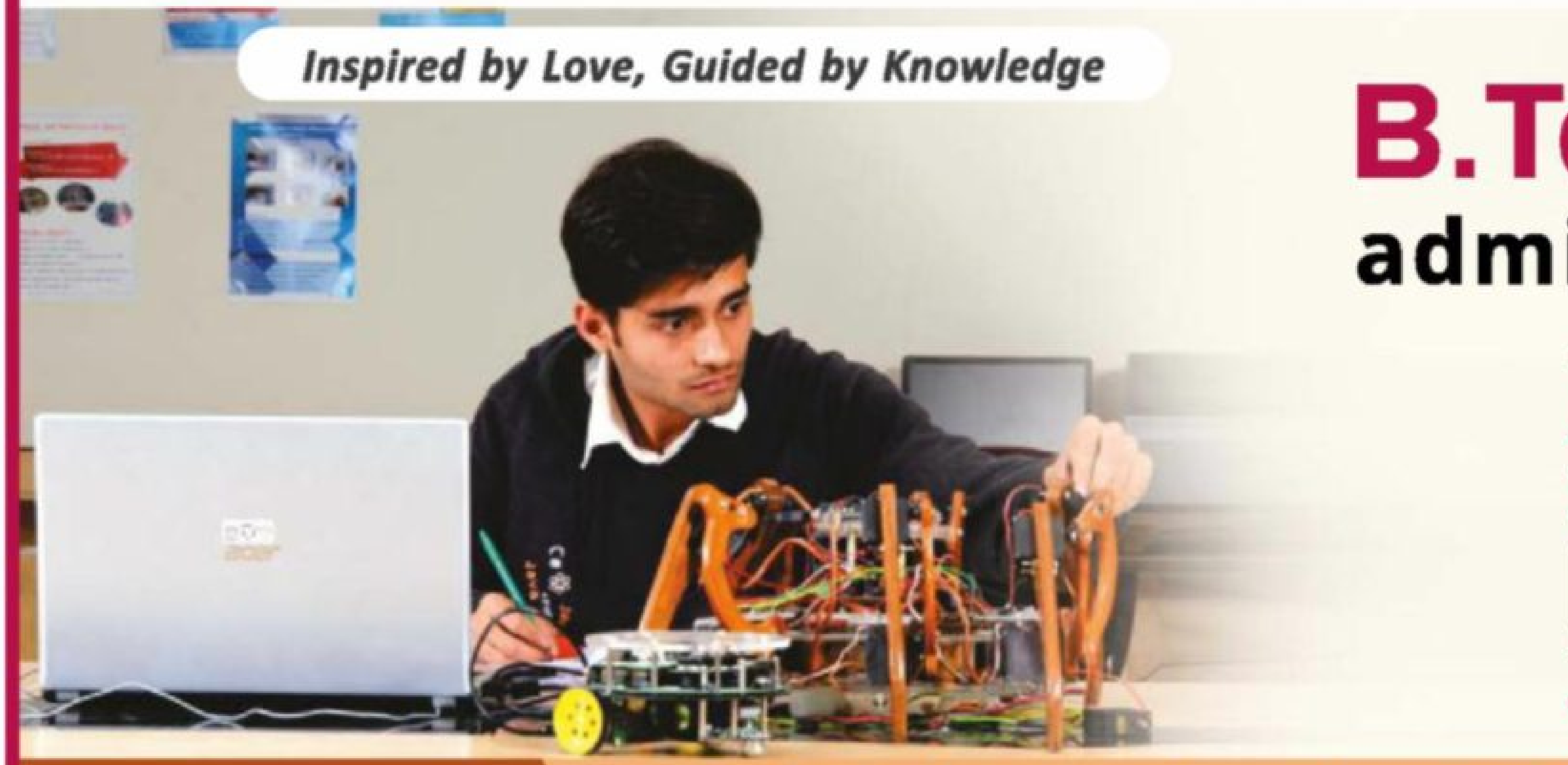

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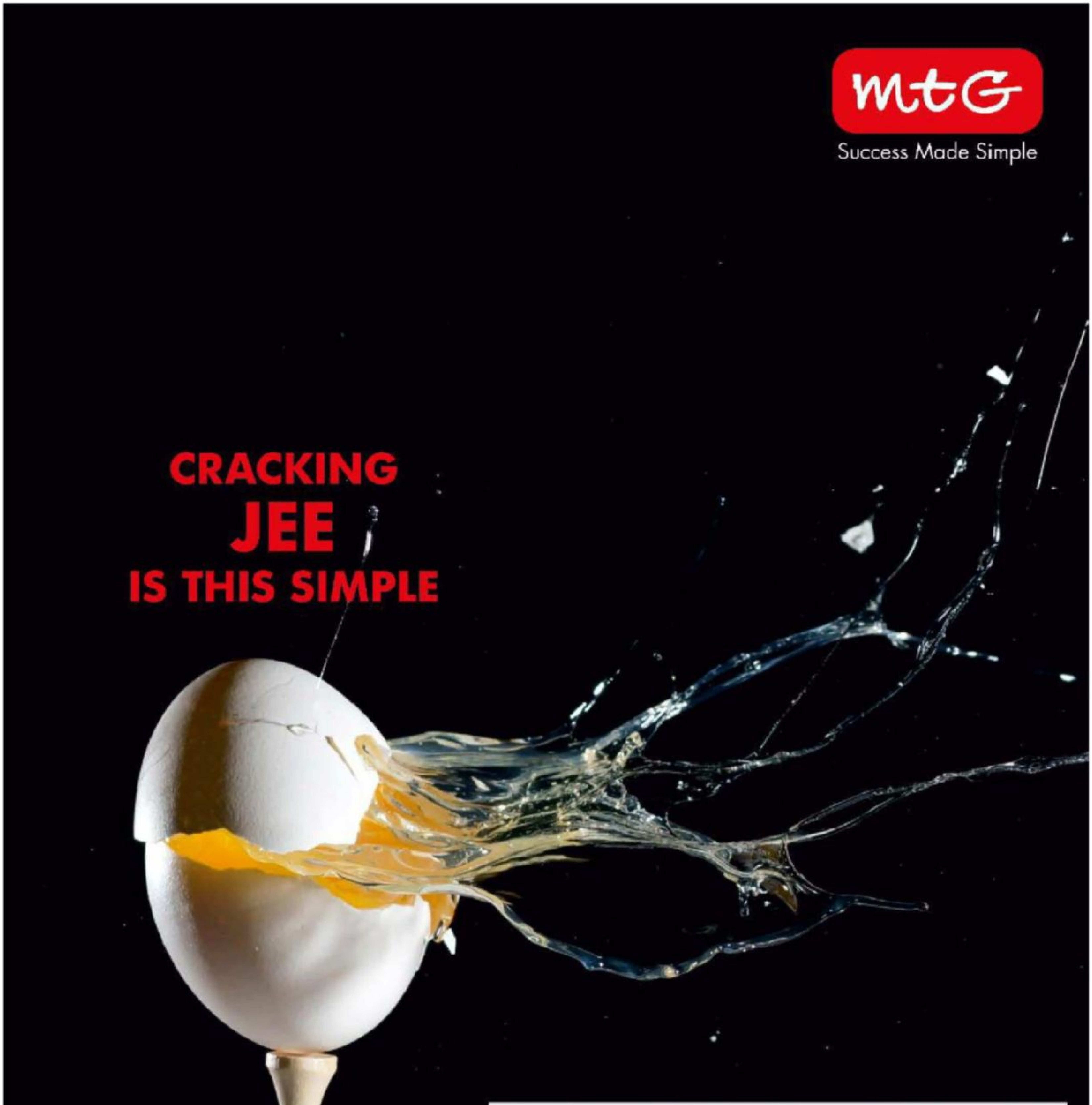
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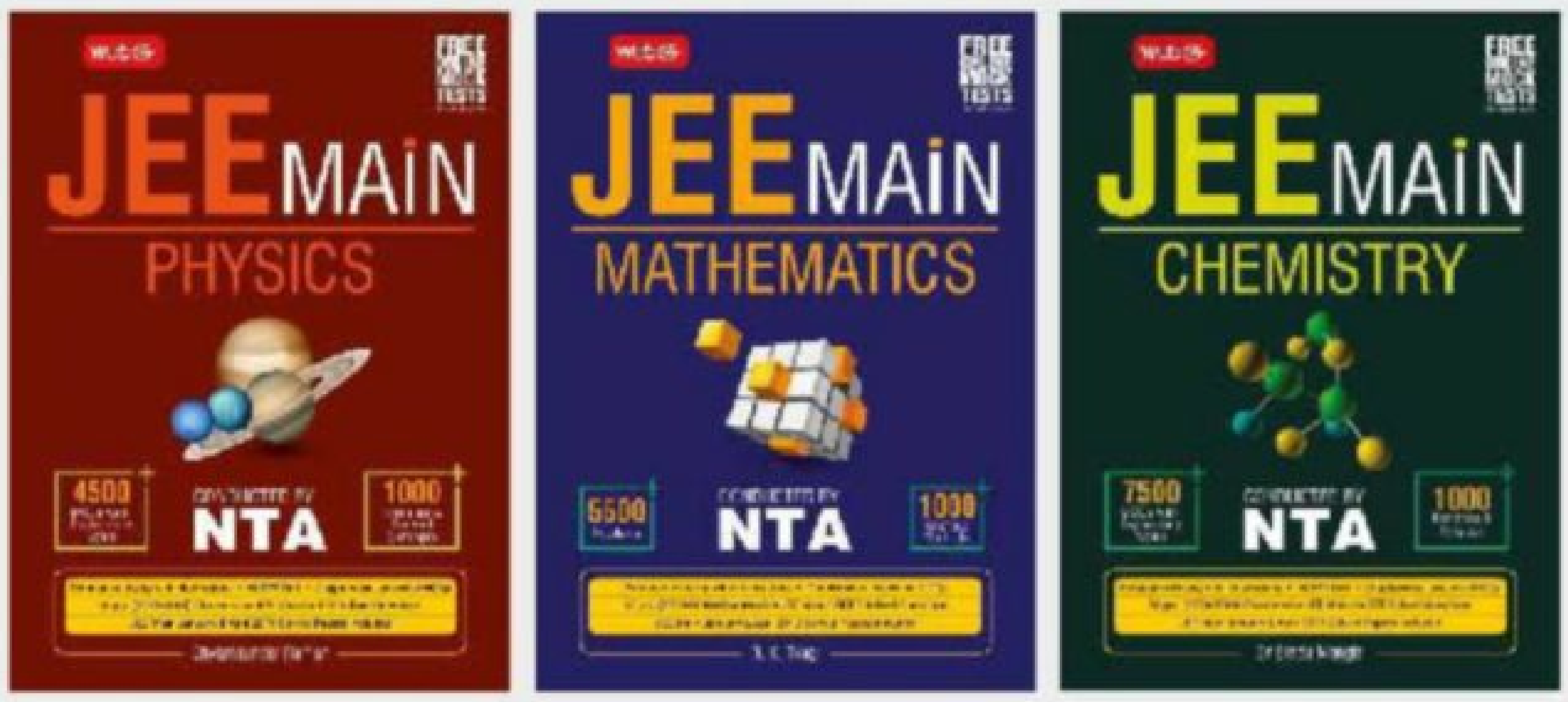
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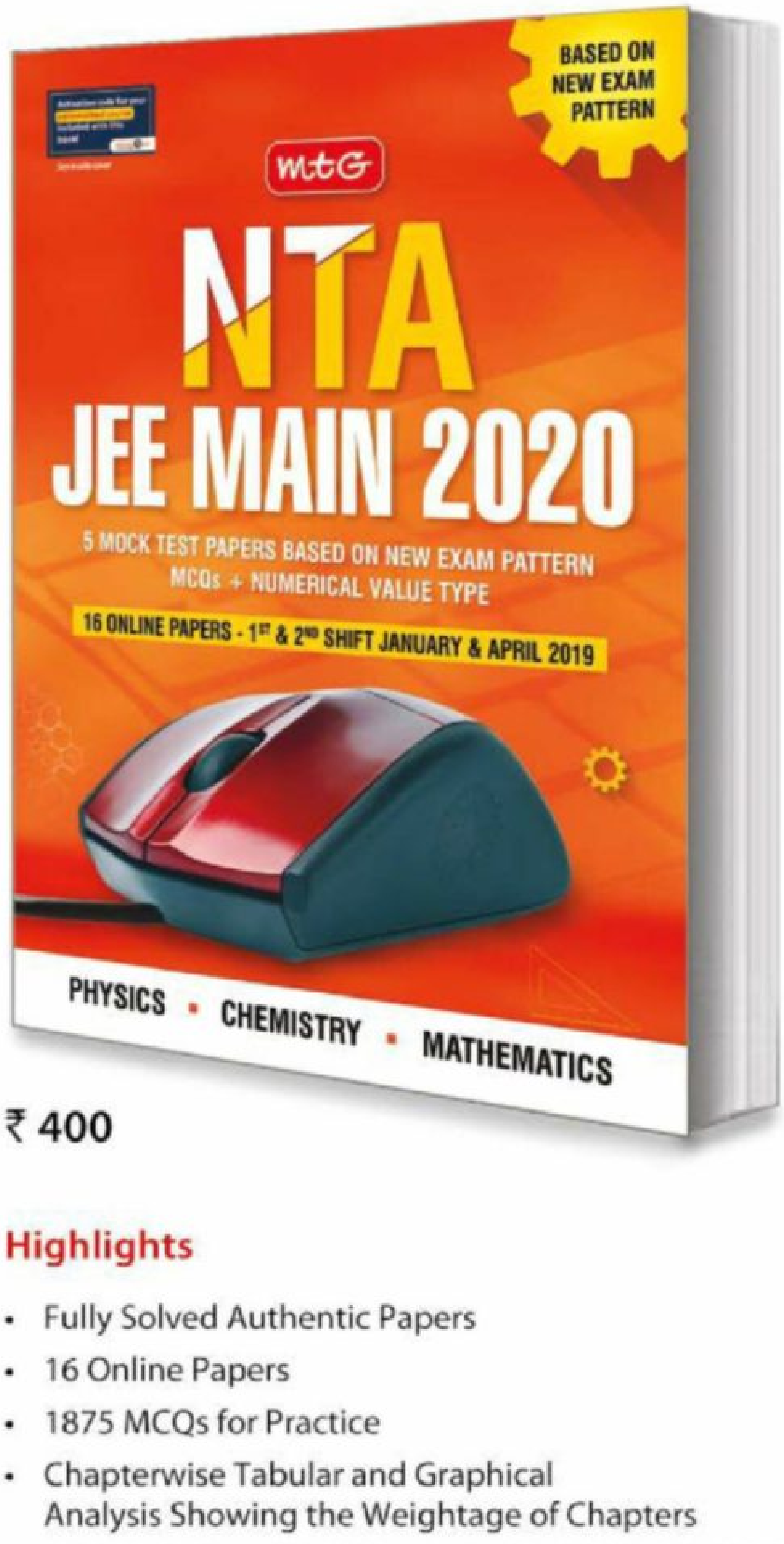
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PROBLEM
Set 204

JEE MAIN

1. The greatest and the least value of $|z_1 + z_2|$ if $z_1 = 24 + 7i$ and $|z_2| = 6$ are respectively

(a) 31, 19 (b) 25, 19
(c) 31, 25 (d) None of these

2. If $I = \int_0^1 (1-x^2)^9 x^9\, dx$, then the sum of the digits of $\frac{1}{I}$ is

(a) 2 (b) 4 (c) 8 (d) 9

3. Let p, q be roots of the equation $x^2 - 4x + A = 0$ and r and s be the roots of the equation $x^2 - 20x + B = 0$. If $p < q < r < s$ are in A.P. then (A, B) is

(a) $(0, -96)$ (b) $(96, 0)$
(c) $(0, 96)$ (d) $(-96, 0)$

4. From a point P tangents are drawn to the circle $x^2 + y^2 = a^2$. If the chord of contact touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the locus of P is

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{a^2}{b^2}$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{b^2}{a^2}$

(c) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = \frac{a^2}{b^2}$ (d) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = \frac{b^2}{a^2}$

5. If events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$, then $P(B / (A \cup \bar{B})) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

JEE ADVANCED

6. The sides of an acute angled triangle are 10, 24, x. The integral values of x are

(a) 22 (b) 23 (c) 24 (d) 25

COMPREHENSION

The vertices of a tetrahedron are $O(0, 0, 0)$, $A(1, 2, 3)$, $B(2, 3, 1)$, $C(3, 1, 2)$.

7. The distance of the vertex A from the edge BC is

(a) $\frac{5\sqrt{3}}{6}$ (b) $\frac{6\sqrt{3}}{5}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

8. The distance of the vertex A from the face OBC is

(a) $\frac{5\sqrt{3}}{6}$ (b) $\frac{6\sqrt{3}}{5}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

NUMERICAL VALUE TYPE

9. Let $C_r = \binom{10}{r}$ and $S = \frac{C_0}{5} - \frac{C_1}{6} + \frac{C_2}{7} - \frac{C_3}{8} + \ldots + \frac{C_{10}}{15}$.

The number of prime factors of $\frac{1}{S}$ is

MATRIX MATCH

10. The letters of the word CALCULUS are used to form 3, 4, 5, 6-letter words.

	Column-I		Column-II
P.	The sum of the digits of the number of 3-letter words is	1.	3
Q.	The sum of the digits of the number of 4-letter words is	2.	12
R.	The sum of the digits of the number of 5-letter words is	3.	15
S.	The sum of the digits of the number of 6-letter words is	4.	18

	P	Q	R	S
(a)	3	4	1	2
(b)	4	1	2	3
(c)	3	2	1	4
(d)	2	3	4	1

◆

See Solution Set of Maths Musing 203 on page no. 48

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Knowledge Quiz - 25

Find integers m, n with
$m^2 + (m + 1)^2 = n^4 + (n + 1)^4$

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Unit 6

Straight Lines / Conic Sections

STRAIGHT LINE

Straight line is a curve such that every point on the line segment joining any two points on it entirely lies on it.

Slope/Gradient of a Line

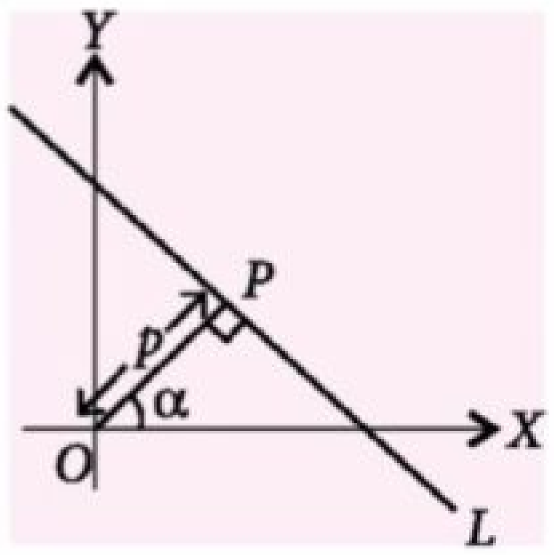
If a line makes an angle α with the positive direction of x-axis then $\tan\alpha$ is said to be the slope or gradient of the line. We generally denote it by m.

$$\therefore \quad m = \tan\alpha = \frac{dy}{dx}$$

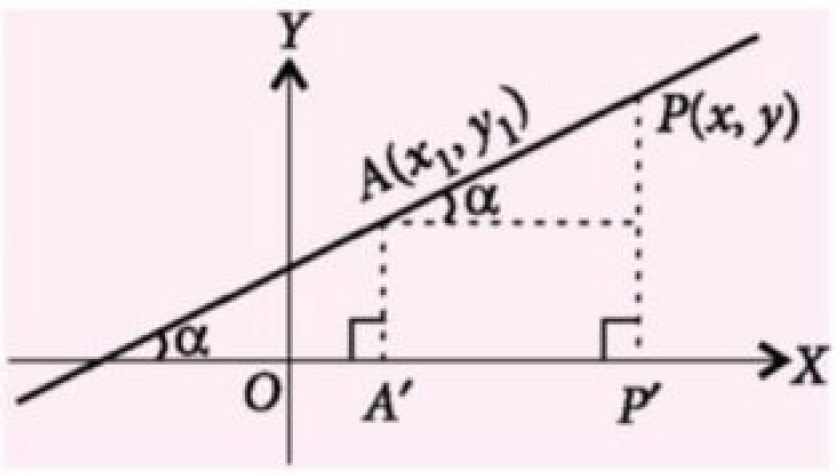
- The slope of a line parallel to x-axis is $m = \tan 0° = 0$.
- The slope of a line parallel to y-axis is not defined.
- If a line is equally inclined with the axes then its slope is 1 or –1.
- If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points, then slope of AB is $\dfrac{y_2 - y_1}{x_2 - x_1} = \tan\alpha = \dfrac{dy}{dx}$
- Slope of a line $ax + by + c = 0$, $b \neq 0$ is $-\dfrac{a}{b} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } y}$

Equation of Straight Line in Different Forms

- Equation of line parallel to x-axis is $y = \pm a$ $(a \in R)$
- Equation of line parallel to y-axis is $x = \pm b$ $(b \in R)$
- **Point slope form :** Equation of line L having slope m and passing through a point (x_1, y_1) is
$$y - y_1 = m(x - x_1)$$
- **Two point form :** Equation of line L passing through $P(x_1, y_1)$, $Q(x_2, y_2)$ is
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
- **Slope intercept form :** If a line L has a slope m and its intercept on y-axis is c, then equation of line L is given by $y = mx + c$.
- **Intercept form :** Let a line L makes intercept a on x-axis and b on y-axis, then its equation is given by
$$\frac{x}{a} + \frac{y}{b} = 1.$$
- **Normal form :** Let length of perpendicular from $O(0, 0)$ to the line L is $OP = p$ and α be the angle made by OP with the +ve direction of x-axis then equation of line L is given by $x\cos\alpha + y\sin\alpha = p$



- **Parametric form :** Equation of line in parametric form is $\dfrac{x - x_1}{\cos\alpha} = \dfrac{y - y_1}{\sin\alpha} = \pm r$



where r is the distance between the point $P(x, y)$ and $A(x_1, y_1)$. The general point on the line is $x = x_1 + r\cos\alpha$, $y = y_1 + r\sin\alpha$.

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi

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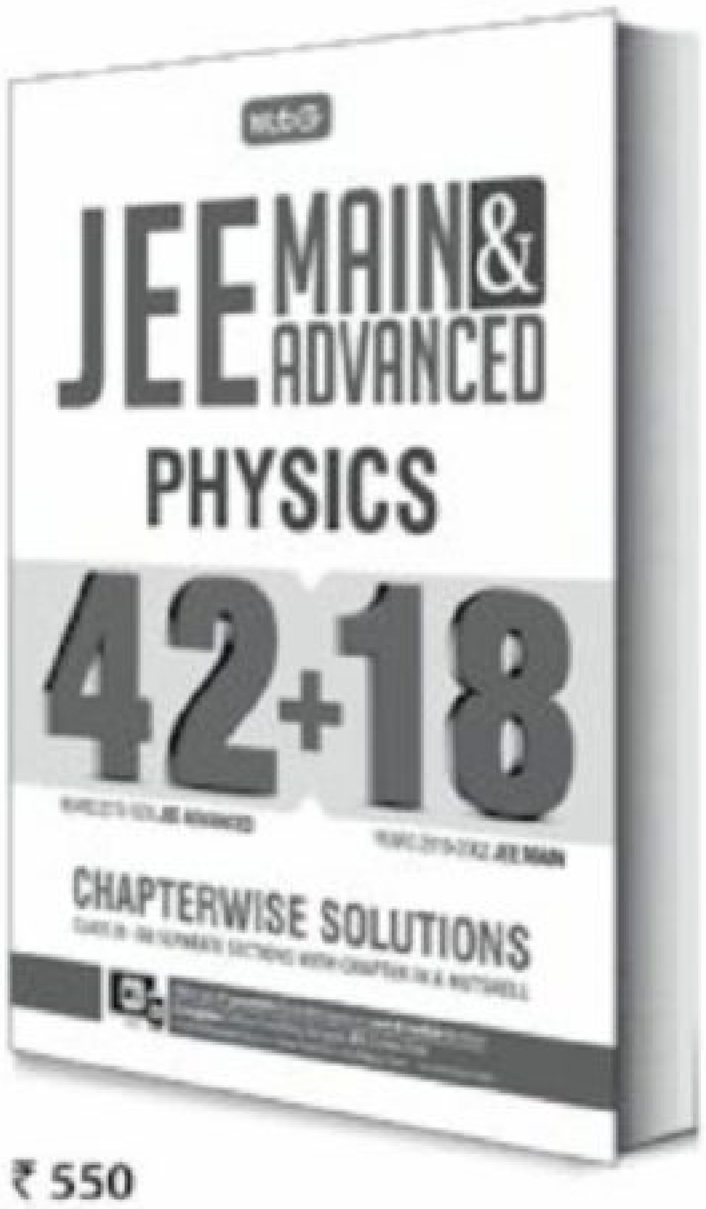
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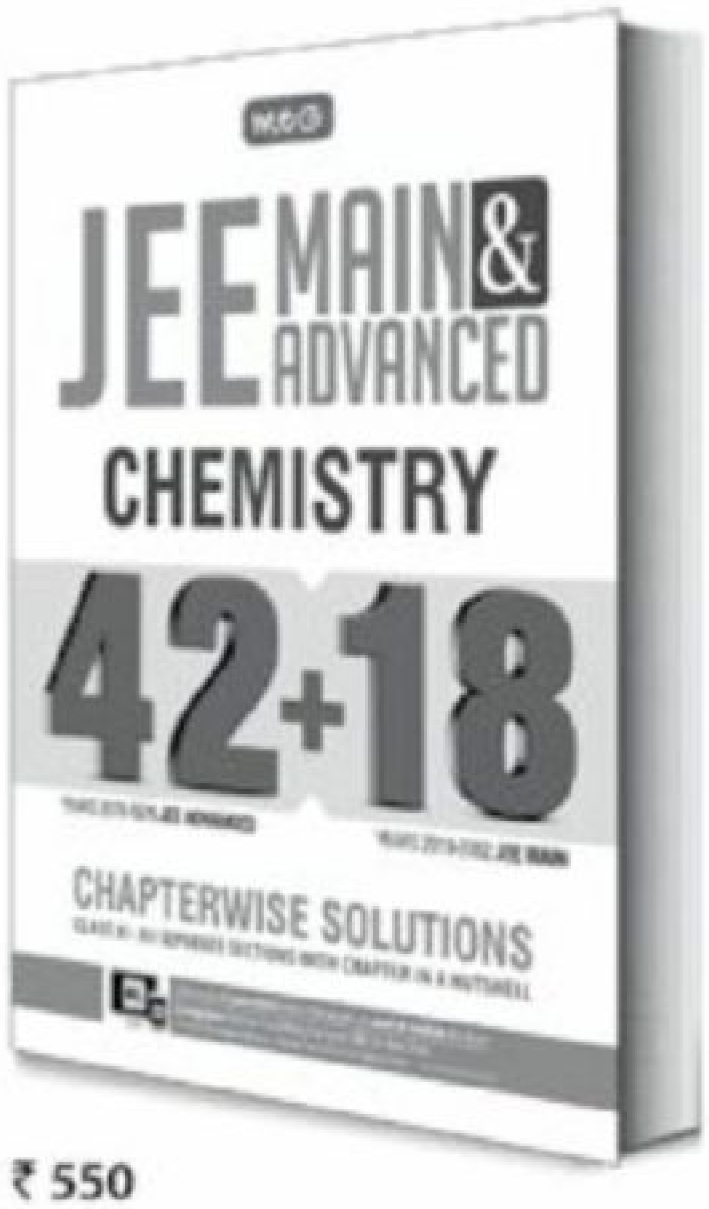
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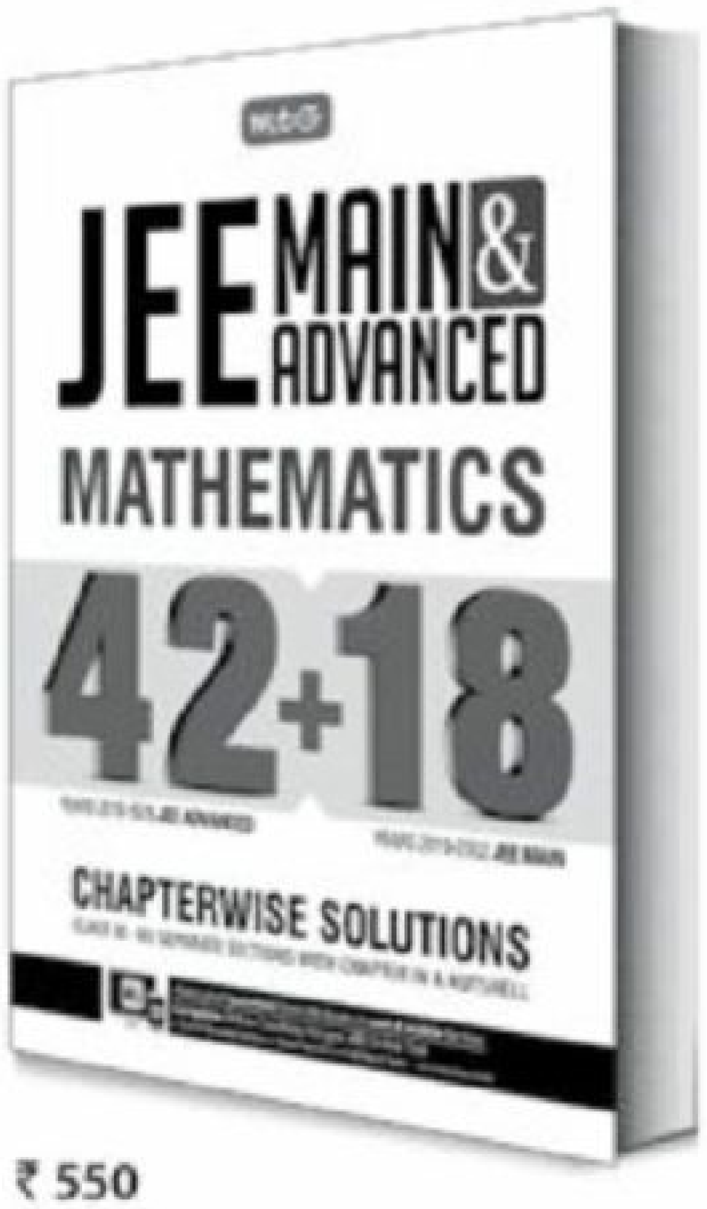
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Equation of Line Parallel and Perpendicular to a Given Line

Let $ax + by + c = 0$ be a given line then the equation of parallel line to the given line is
$ax + by + \lambda = 0$, where $\lambda \neq c$ is any arbitrary constant.
The equation of a line perpendicular to $ax + by + c = 0$ is given by $bx - ay + \lambda = 0$, where $\lambda \neq c$ is any arbitrary constant.

General Equation of Family of Lines Through the Intersection of Two Given Lines

Let the two lines are $L_1 : a_1x + b_1y + c_1 = 0$ and $L_2 : a_2x + b_2y + c_2 = 0$, then the equation of family of lines passing through the intersection of L_1 and L_2 is given by $L_1 + \lambda L_2 = 0$ or $L_2 + \lambda L_1 = 0$
i.e., $(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$
or $(a_2x + b_2y + c_2) + \lambda(a_1x + b_1y + c_1) = 0$, where λ is a parameter.

Angle between Two Lines

Let α be the angle between the lines $L_1 = 0$ and $L_2 = 0$ where $L_1 : y = m_1x + c_1$ and $L_2 : y = m_2x + c_2$, then

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Note : If m_1 and m_2 are slope of two

- parallel lines, then $m_1 = m_2$.
- perpendicular lines, then $m_1m_2 = -1$.

Equation of Straight Lines Through a Given Point Which Makes the Given Angle with a Given Line

Let $P(x_1, y_1)$ be a given point and $y = mx + c$ be a given straight line. Let α be the angle made by the line whose equation is to be determine with the given line $y = mx + c$, then required equation of line is

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Distance of a Point from a Straight Line

Let $P(x_1, y_1)$ be any point at a distance d from the line $L : ax + by + c = 0$, then $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between the Parallel Lines

$L_1 : ax + by + c_1 = 0$ and $L_2 : ax + by + c_2 = 0$, then distance between L_1 and L_2 is $d = \dfrac{\alpha}{\sqrt{a^2 + b^2}}$, where

(i) $\alpha = |c_1 - c_2|$ if they are on the same side of the origin.
(ii) $\alpha = |c_1| + |c_2|$, if origin lies between L_1 and L_2.

Concurrency of Lines

Three or more lines are said to be concurrent if they meet/intersect at the same point. Let the three lines be $L_1 : a_1x + b_1y + c_1 = 0$, $L_2 : a_2x + b_2y + c_2 = 0$ and $L_3 : a_3x + b_3y + c_3 = 0$.

These lines will be concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$

- The condition for the lines $L = 0$, $M = 0$ and $N = 0$ to be concurrent is that three constants a, b, c (not all zero at the same time) can be obtained such that $aL + bM + cN = 0$.

Position of a Point with respect to a Given Line

Let $L : ax + by + c = 0$ be given line and $P(x_1, y_1)$ be the point whose position is to be determined.

- If the same sign is obtained by putting $x = x_1$ and $y = y_1$ and $x = 0 = y$ in the given equation, then the point $P(x_1, y_1)$ lies on the side of the origin.
- If the opposite sign is obtained by putting $x = x_1$, $y = y_1$ and $x = 0 = y$ in the equation of the line then, the point $P(x_1, y_1)$ lies on the opposite side of the origin.

Position of Two Points with respect to a Given Line

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points and $L = ax + by + c = 0$ be equation of line. Then

(a) two points lie on the same side of the line if

$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$

(b) two points lie on the opposite sides of the line if

$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$$

Mirror Image of a Point with respect to a Given Line

Let $L : ax + by + c = 0$ be the equation of line. Let (x_2, y_2) be the mirror image of (x_1, y_1), then

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

Note : If (x_2, y_2) be the foot of the perpendicular from (x_1, y_1) to the line $ax + by + c = 0$, then

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

PAIR OF STRAIGHT LINES

Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ be two lines then their joint equation is

$$(a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0 \qquad \text{...(i)}$$

Now, (i) can be written in the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \qquad \text{...(ii)}$$

It is known as general equation of second degree.

Note : (1) If θ is the angle between the lines then

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b} = \pm\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- ➢ If $a + b$ = coefficient of x^2 + coefficient of $y^2 = 0$, then lines are perpendicular.

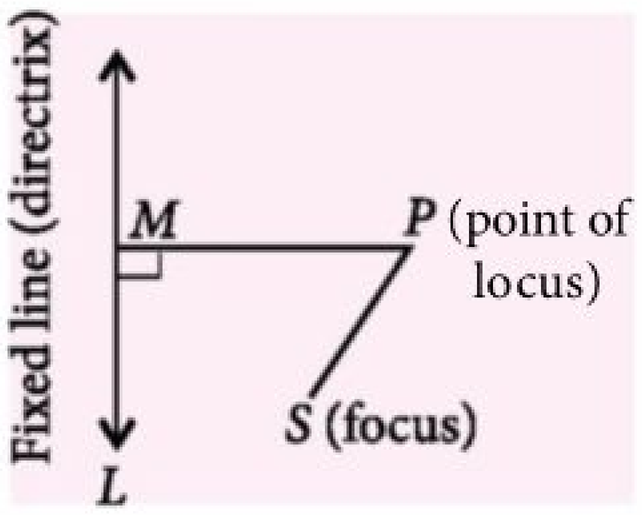
(2) Let us consider the equation given in (ii) and say

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

If $\Delta = 0$ *i.e.* $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and $h^2 > ab$ then (ii) represent intersecting lines and their point of intersection is given by

$$\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right)$$

CONIC SECTION



- A conic is the locus of a point (P) which moves in such a way that its distance from a fixed point (S) always bears a constant ratio to its distance from a fixed straight line (L). The fixed point (S) is called focus and the fixed straight line is called directrix L of the conic. The constant ratio is called eccentricity which is denoted by e.

$$\therefore \quad e = \frac{PS}{PM} \qquad \text{...(i)}$$

- If $0 < e < 1$, the conic is an ellipse.
- If $e = 1$, the conic is a parabola.
- If $e > 1$, the conic is a hyperbola.
- Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ...(ii) is a general equation of second degree.

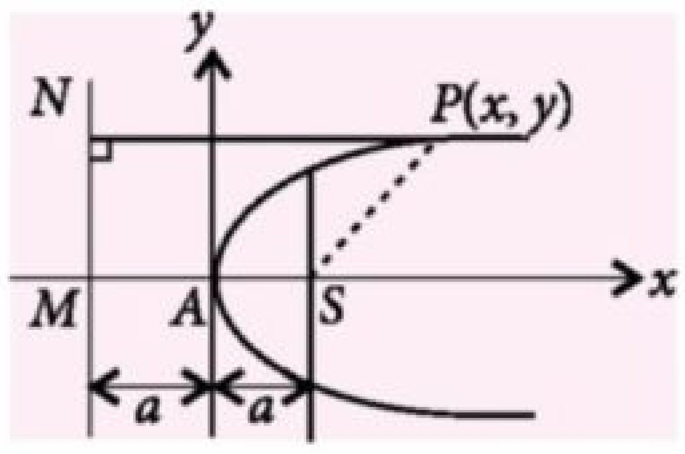
$$\text{Let } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(i) If $\Delta \neq 0$ and $h^2 = ab$, then (ii) represents a parabola.

(ii) If $\Delta \neq 0$ and $h^2 < ab$, then (ii) represents an ellipse.

(iii) If $\Delta \neq 0$ and $h^2 > ab$, then (ii) represents a hyperbola and in addition to if $a + b = 0$, then hyperbola is called rectangular hyperbola.

PARABOLA



Let S be the focus (fixed point) and MN be the directrix (fixed straight line). Let SM be perpendicular to MN and A be the mid point of SM. Assuming A as origin, AS as x-axis. $MA = AS = a$ and equation of directrix is $x = -a$ and co-ordinates of focus S are $(a, 0)$. So, $y^2 = 4ax$ is the standard equation of parabola.

- **Axis of parabola :** A straight line passing through the focus and perpendicular to the directrix is called axis of parabola.
- **Vertex of parabola :** The point of intersection of parabola and its axis is called its vertex.
- **Latus rectum or double ordinate :** A chord passing through the focus and perpendicular to the axis is called latus rectum. The length of latus rectum is $4a$. The coordinates of extremities of latus rectum of $y^2 = 4ax$ are $(a, 2a)$ and $(a, -2a)$.

Parametric Equations of Parabola

For any parameter t, $x = at^2$ and $y = 2at$ are called parametric equations of parabola $y^2 = 4ax$.

Tangents and Normals to the Parabola

- **Different forms of tangent lines to parabola $y^2 = 4ax$ at a given point**

Point	Equation of tangent
(x_1, y_1)	$yy_1 = 2a(x + x_1)$ (point form)
$(at^2, 2at)$	$ty = x + at^2$ (parametric form)
$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y = mx + \frac{a}{m}$ (slope form)

- **Condition of tangency :** Let equation of parabola is $y^2 = 4ax$. Then,

Line	Condition
$y = mx + c$	$c = \frac{a}{m}$
$x\cos\alpha + y\sin\alpha = p$	$p = -a \tan\alpha \sin\alpha$
$lx + my + n = 0$	$ln = am^2$

- **Pair of tangents from a point to parabola :** Let $y^2 = 4ax$ be the equation of parabola and $A(x_1, y_1)$ be a point lying outside the parabola.

Let $S = y^2 - 4ax$, $S_1 = y_1^2 - 4ax_1$ and $T = yy_1 - 2a(x + x_1)$

Then equation of pair of tangents is given by $SS_1 = T^2$.

- **Point of intersection of tangents at t_1 and t_2 :** Let $y^2 = 4ax$ be the equation of parabola and $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$ are any two points on it. Then the coordinates of point of intersection of tangents at t_1 and t_2 is given by $(at_1t_2, a(t_1 + t_2))$.
  Note : If tangents at t_1 and t_2 are perpendicular, then $t_1t_2 = -1$.
- **Different forms of equation of normal to the parabola $y^2 = 4ax$ at a given point**

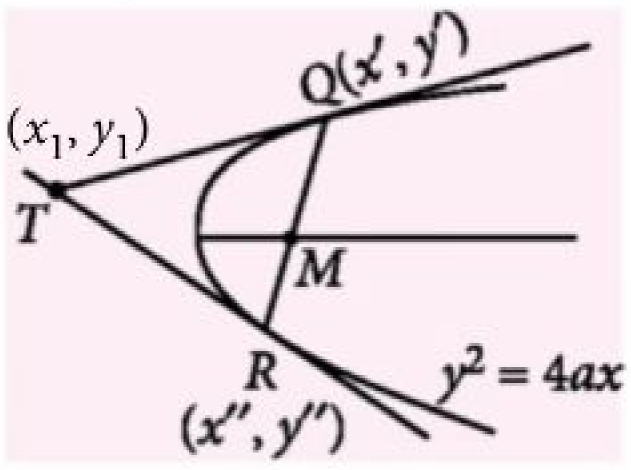
Point	Equation of normal
(x_1, y_1)	$y - y_1 = -\frac{y_1}{2a}(x - x_1)$ (one point form)
$(at^2, 2at)$	$y + tx = at^3 + 2at$ (parametric form)
$(am^2, -2am)$	$y = mx - 2am - am^3$ (point slope form)

- **Co-normal points :** The points on the curve at which the normal passes through a common point are called co-normal points. Sometimes co-normal points are said to be feet of the normals.

Equation of Chord of Contact of Tangents to a Parabola

Equation of chord of contact QR is given by

$$yy_1 = 2a(x + x_1)$$



Chord of a Parabola with Specific Mid-point

Equation of chord of parabola bisected at a given point (x_1, y_1) is $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$.

Equation of Chord Joining Two Points on a Parabola

- Let $P(x_1, y_1)$, $Q(x_2, y_2)$ be any two points on the parabola $y^2 = 4ax$. Then equation of the chord PQ is
  $$y - y_1 = \frac{4a}{(y_2 + y_1)}(x - x_1)$$
- If the point be $P(t_1)$ and $Q(t_2)$ then the equation of the chord PQ is $y(t_1 + t_2) = 2(x + at_1t_2)$

Note : If PQ passes through the focus of the parabola, then $t_1t_2 = -1$.

Diameter of a Parabola

The locus of the midpoints of a system of parallel chords of a parabola is called its diameter.

Director Circle of Parabola

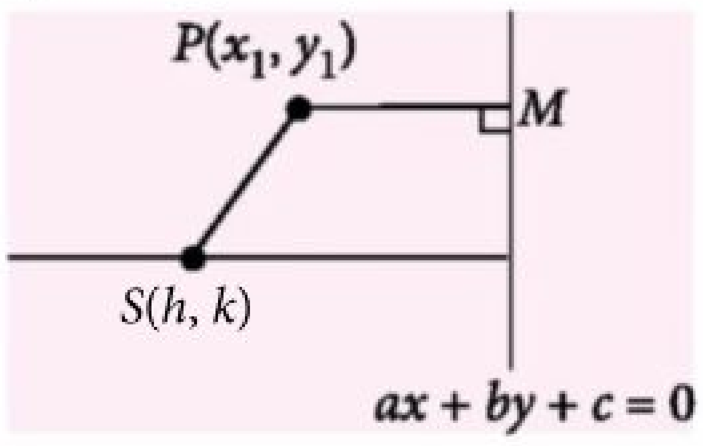
The locus of the point of intersection of perpendicular tangents to the parabola is called its director circle. The director circle of parabola $y^2 = 4ax$ is its directrix *i.e.*, $x = -a$.

ELLIPSE

Let the focus of an ellipse be $S(h, k)$ and equation of its directrix line is $ax + by + c = 0$. Let e be the eccentricity and $P(x_1, y_1)$ be any point on the ellipse then by definition of the ellipse

$$SP = ePM$$

$$\Rightarrow (x_1 - h)^2 + (y_1 - k)^2 = \frac{e^2(ax_1 + by_1 + c)^2}{a^2 + b^2}$$

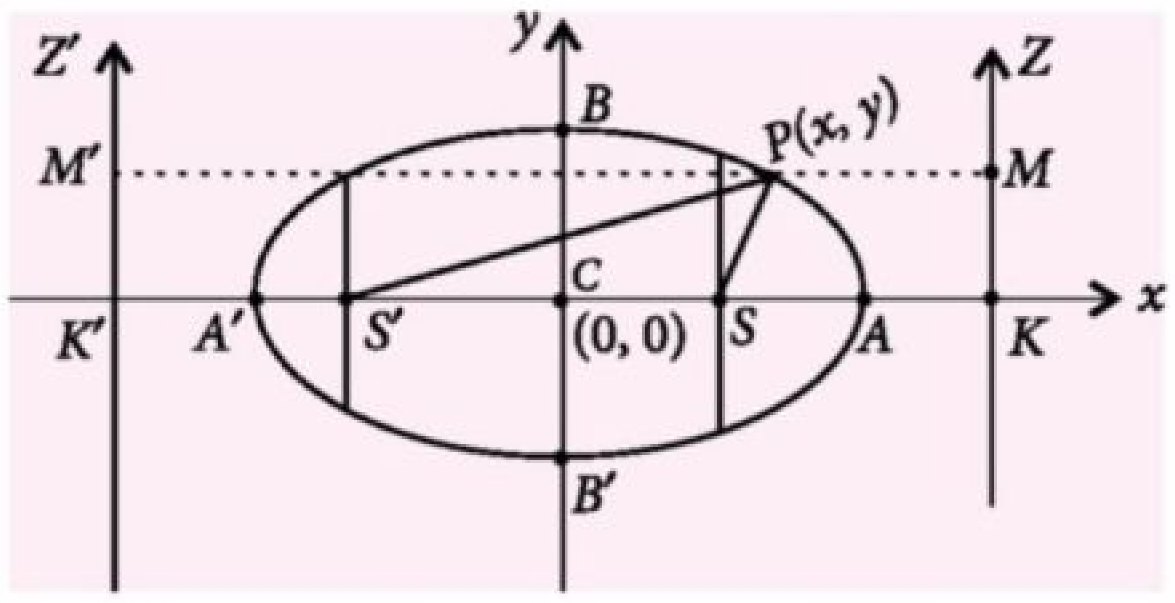


Standard Equation of an Ellipse

Let $P(x, y)$ be any point then the standard equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } b^2 = a^2(1 - e^2).$$

Various Terms of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$

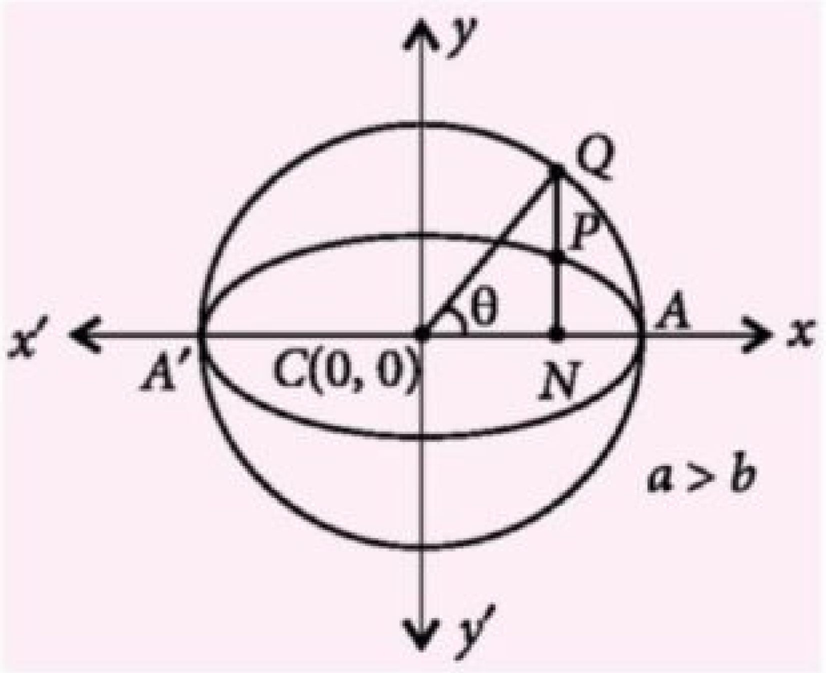


- Vertices are $A(a, 0)$, $A'(-a, 0)$. Line joining the foci S and S' is called axis of the ellipse.
- The distance $AA' = 2a$ = length of major axis.
- The distance $BB' = 2b$ = length of minor axis.
- The points $S(ae, 0)$, $S'(-ae, 0)$ are foci of ellipse.
- ZK and $Z'K'$ are two directrices of the ellipse and $x = \pm\frac{a}{e}$ are the equations of directrices.

- The point which bisect the every chord passing through it, is called centre. The centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is origin. So, the mid-point of AA' is the centre of the ellipse.
- Eccentricity, (e) is given by $e^2=\left(1-\frac{b^2}{a^2}\right)$.
- **Latus rectum :** A chord through either focus and perpendicular to the major axis is said to be latus rectum. Length of latus rectum is $\frac{2b^2}{a}$.
- **Focal chord :** Any chord through either focus is said to be focal chord.

Auxiliary Circle, Eccentric Angle and Parametric Equations of an Ellipse

- The circle described on the major axis of an ellipse as diameter is called auxiliary circle of the ellipse. The equation of auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a>b)$ is $x^2+y^2=a^2$.
- If P be any point on the ellipse, Q be any point on the auxiliary circle such that QPN is perpendicular to major axis, *i.e.*, Q lies on the ordinate produced of the point P.



Then $\angle ACQ=\theta$ is called eccentric angle.
If θ is the eccentric angle of any point $P(\theta)$ then $x=a\cos\theta$ and $y=b\sin\theta$ are known as parametric equations of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Equation of Chord of the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

- The equation of chord joining $P(\theta)$ and $Q(\phi)$ is $\frac{x}{a}\cos\frac{\theta+\phi}{2}+\frac{y}{b}\sin\frac{\theta+\phi}{2}=\cos\frac{\theta-\phi}{2}$.
- Equation of chord of the ellipse bisected at $P(x_1, y_1)$ is given by $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$.

Equation of Tangent to the Ellipse

Let the equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

- **Point form :** Equation of tangent at $P(x_1, y_1)$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.
- **Parametric form :** Equation of tangent at $P(\theta)$ is given by $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$.
- **Slope form :** Equation of tangent to the ellipse is given by $y=mx\pm\sqrt{a^2m^2+b^2}$.
- Equation of pair of tangents from $P(x_1, y_1)$ to the ellipse is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1\right)=\left(\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1\right)^2.$$

Equation of Normal to the Ellipse

Let the equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

- **Point form :** $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$
- **Parametric form :** $ax\sec\theta-by\,\text{cosec}\,\theta=a^2-b^2$
- **Slope form :** $y=mx-\frac{m(a^2-b^2)}{\sqrt{a^2+m^2b^2}}$

Point of Contact and Condition of Tangency of Lines to the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation of line	Point of contact	Condition of tangency
$y=mx+c$	$\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$	$c^2=a^2m^2+b^2$
$lx+my+n=0$	$\left(\frac{a^2l}{n}, \frac{b^2m}{n}\right)$	$n^2=a^2l^2+b^2m^2$
$x\cos\alpha+y\sin\alpha=p$	$\left(\frac{p^2-b^2\sin^2\alpha}{p\cos\alpha}, \frac{b^2\sin\alpha}{p}\right)$	$p^2=a^2\cos^2\alpha+b^2\sin^2\alpha$

Director Circle of an Ellipse

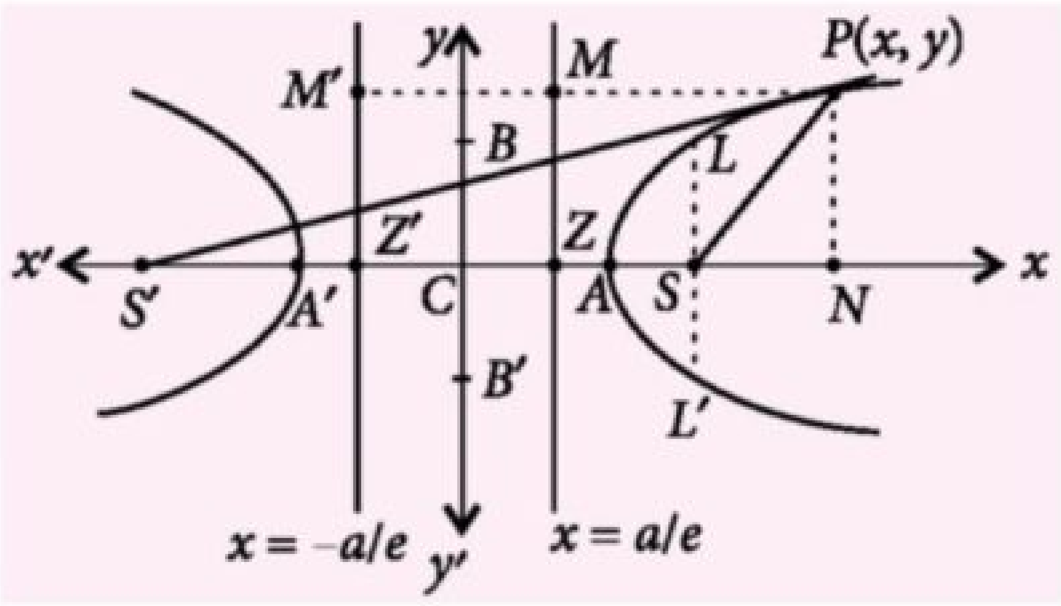
The equation of director circle of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2 + y^2 = a^2 + b^2$.

HYPERBOLA

Let $P(x, y)$ be any point, then standard equation of the hyperbola is given by $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, where $b^2=a^2(e^2-1)$.

Various Terms of Hyperbola

- The straight line AA' is called **transverse axis.** The points $A(a, 0)$, $A'(-a, 0)$ are called **vertices** and $AA' = 2a$ is the length of the transverse axis.
- The straight line BB' is called conjugate axis and $BB' = 2b$ is the length of conjugate axis with $B(b, 0)$ and $B'(-b, 0)$.



- C is the midpoint of AA' and BB', the co-ordinates of C are $(0, 0)$ and C is called centre of hyperbola.
- $S(ae, 0)$ and $S'(ae, 0)$ are the foci of the hyperbola.
- If $P(x, y)$ lies on the hyperbola then PS and PS' are called focal distances.
- The chord LL' through either focus and perpendicular to the transverse axis is called latus rectum of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $(a>b)$. The co-ordinates of L and L' are $\left(ae, \frac{b^2}{a}\right)$ and $\left(ae, \frac{-b^2}{a}\right)$ respectively.

  Length of latus rectum is equal to $\frac{2b^2}{a}$.

Asymptotes of Hyperbola

- If the length of $\perp$'r drawn from a point on the hyperbola to a straight line tends to zero as the point moves to infinity, then the straight line is called asymptote. The equations of asymptotes to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is given by $\frac{x}{a}\pm\frac{y}{b}=0$.

Conjugate Hyperbola

The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axes of the given hyperbola is called conjugate hyperbola of the given hyperbola.

- If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be given hyperbola then its conjugate hyperbola is given by $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$.
- If e and e' be respective eccentricities of a hyperbola and its conjugate then $\frac{1}{e^2}+\frac{1}{e'^2}=1$.
- The equation of asymptotes of a hyperbola and its conjugate are same.

Rectangular Hyperbola

A hyperbola whose asymptotes are at right angles to each other is called a rectangular hyperbola and its eccentricity is $\sqrt{2}$.

- The equation of rectangular hyperbola is $x^2 - y^2 = a^2$.
- If the axes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are rotated through an angle $\pi/4$ about the same origin, then equation of rectangular hyperbola reduces to

  $xy=\frac{a^2}{2}$ or $xy=c^2$, where $c^2=\frac{a^2}{2}$.
- For rectangular hyperbola $xy = c^2$, the asymptotes are coordinates axis and joint equation of asymptotes in this case is $xy = 0$.
- The conjugate hyperbola of $xy = c^2$ is $xy = -c^2$.
- The parametric equation of rectangular hyperbola $xy=c^2\left(=\frac{a^2}{2}\right)$ is given by $x = ct$ and $y = c/t$.

Equation of Tangent at a Point to the Hyperbola

- Equation of tangent to the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (x_1, y_1) is $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$.
- Equation of tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $(a\sec\theta, b\tan\theta)$ is $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$.

- For rectangular hyperbola, $xy = c^2$, equation of tangent at (x_1, y_1) is $\frac{x}{x_1}+\frac{y}{y_1}=2.$

Pair of Tangents Drawn from a Point Outside the Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

From a point outside the hyperbola, pair of tangents can be drawn and their joint equation is given by $SS_1 = T^2$, where

$$S=\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ S_1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}=1,\ T=\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1.$$

Equation of Normal to the Hyperbola

- Equation of normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at a point (x_1, y_1) is $\frac{a^2(x-x_1)}{x_1}+\frac{b^2(y-y_1)}{y_1}=0.$
- At a point $P(a\sec\theta, b\tan\theta)$, the equation of normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $ax\cos\theta + by\cot\theta = a^2 + b^2$.
- Equation of normal to $xy = c^2$ at $(ct, c/t)$ is given by $xt^3 - yt = c(t^4 - 1)$.

Director Circle of Hyperbola

The locus of the point of intersection of two perpendicular tangents to the hyperbola is called director circle of hyperbola. The equation of director circle of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is given by $x^2 + y^2 = a^2 - b^2$.

PROBLEMS

Single Option Correct Type

1. A point (12, 2) assumes the following three transformations successively.

(1) Reflection about the line $y = x$.

(2) Translation through a distance 3 units along the positive direction of x-axis.

(3) Rotation through an angle 45° about the origin in the anticlockwise direction.

Then the final position of the point is

(a) $\left(\frac{7}{\sqrt{2}}, \frac{17}{\sqrt{2}}\right)$ (b) $\left(\frac{-7}{\sqrt{2}}, \frac{17}{\sqrt{2}}\right)$

(c) $(-7\sqrt{2}, 17\sqrt{2})$ (d) $\left(\frac{7}{\sqrt{2}}, \frac{-17}{\sqrt{2}}\right)$

2. If P_1, P_2, P_3 be lengths of perpendiculars from the points $(m^2, 2m)$, $(mm', m + m')$ and $(m'^2, 2m')$ on the line $x\cos\theta + y\sin\theta + \sin\theta\tan\theta = 0$, then P_1, P_2, P_3 are in

(a) G.P. (b) A.P.

(c) H.P. (d) none of these

3. The angle of intersection of the curve $y=(3+x)^{\cos x}-\frac{2}{3}\sin x$ and the straight line $x + 12y - 36 = 0$ at a point (α, β), where α and β are integers is θ, then the value of $\tan\theta$ equals to

(a) 35/36 (b) 5/7 (c) 3/7 (d) 15/36

4. The mirror image of the line $2x - 3y - 5 = 0$ in the line $x + 2y + 3 = 0$ is

(a) $x + 18y + 1 = 0$ (b) $18x + y - 1 = 0$

(c) $18x - y - 1 = 0$ (d) none of these

5. When the origin is shifted to (2, –3) the equation $y^2 + 8x + 6y - 7 = 0$ reduces to $y^2 = 4ax$. The value of 'a' is equal to

(a) 4 (b) 2 (c) –2 (d) –4

6. If the lines joining the origin to the common points of the curve $(x - 4)^2 + (y - 5)^2 = m^2$ and the straight line $5x + 4y - 40 = 0$ are at right angle, then the value of m^2 equals

(a) 20 (b) 25 (c) 36 (d) 41

7. The equation of the chord joining the points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

(a) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ (b) $\frac{x}{x_1^2+x_2^2}+\frac{y}{y_1^2+y_2^2}=1$

(c) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ (d) $\frac{x}{y_1^2+y_2^2}+\frac{y}{x_1^2+x_2^2}=1$

8. The length of latus rectum of the conic whose differential equation is $xdy + ydx = 0$ and passing through the point (4, 9) is

(a) $6\sqrt{2}$ (b) $12\sqrt{2}$ (c) 12 (d) 72

EXAM ALERT 2020

Exam	Date
JEE Main I	6th to 11th January
WB JEE	2nd February
JEE Main II	3rd to 9th April
JEE Advanced	17th May

9. The equation of hyperbola whose asymptotes are $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ which passes through the point (0, 1) is $12x^2 - 7xy - 12y^2 + 31x + 17y + A = 0$ and the equation of conjugate hyperbola is $12x^2 - 7xy - 12y^2 + ax + by + c = 0$, then the sum of the digits of $a + b + c$ is

(a) 15 (b) 12 (c) 67 (d) 13

10. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\pi/6$, then the eccentricity of conjugate hyperbola is

(a) $4(2+\sqrt{3})$ (b) $\sqrt{2}+\sqrt{6}$

(c) $4(2-\sqrt{3})$ (d) $\sqrt{6}-\sqrt{2}$

More Than One Option(s) Correct Type

11. The equation $f(x, y) = 3x^2 + 10xy + 8y^2 + 14x + 22y + 7 = 0$ represent a hyperbola with/whose

(a) centre at (1, –2)

(b) one of the asymptote is $x + 2y + 3 = 0$

(c) angle between the asymptotes is $\tan^{-1}(2/11)$

(d) equation of conjugate hyperbola is $3x^2 + 10xy + 8y^2 + 14x + 22y + 23 = 0$

12. If the circle $x^2 + y^2 = a^2$ intersect the hyperbola $xy = c^2$ in four points (x_i, y_i), $i = 1, 2, 3, 4$, then

(a) $\sum_{i=1}^{4} x_i = 0$ (b) $\sum_{i=1}^{4} y_i = 0$

(c) $\prod_{i=1}^{4} x_i = c^4$ (d) $\prod_{i=1}^{4} y_i = c^4$

13. If the tangent to the ellipse $9x^2 + 16y^2 = 144$ at a point $P(\theta)$ is normal to circle $x^2 + y^2 - 8x - 6y = 0$, then θ equals

(a) $\pi/6$ (b) $\pi/2$

(c) 0 (d) $-\pi/6$

14. The chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose middle point is $\left(\frac{1}{2}, \frac{2}{5}\right)$ meet the ellipse at $A(x', y')$ and $B(x'', y'')$, then

(a) the equation of the chord is $4x + 5y = 4$

(b) $A(x', y') = \left(4, \frac{-12}{5}\right)$

(c) $B(x'', y'') = \left(-3, \frac{16}{5}\right)$

(d) length of chord AB is $\frac{7\sqrt{41}}{5}$

15. If the equation of directrix of the parabola $x^2 - 8x + 4y + \lambda = 0$ is $y + 2 = 0$, then

(a) $\lambda = 28$

(b) equation of axis of parabola is $x - 4 = 0$

(c) focus of parabola is (4, –4)

(d) vertex of parabola is (4, –3) and equation of tangent at vertex is $y + 3 = 0$

16. Equations $(a - b)x + (b - c)y + (c - a) = 0$ and $(a^3 - b^3)x + (b^3 - c^3)y + (c^3 - a^3) = 0$ will represent the identical lines if

(a) $a + b + c = 0$ (b) $a = b$

(c) $b = c$ (d) $c = a$

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Comprehension Type

Paragraph for Q. No. 17 to 20

The vertices of ΔABC lies on a rectangular hyperbola such that the orthocentre of triangle is (3, 2) and the asymptotes of rectangular hyperbola are parallel to coordinate axis. The two perpendicular tangents of the hyperbola intersect at the point (1, 1).

17. Equation of asymptotes of the hyperbola is

(a) $(x - 1)(y - 1) + \lambda = 0$

(b) $(x - 1)(y - 1) = 0$

(c) $xy + 1 = x - y$

(d) $(x + 1)(y + 1) = 0$

18. The equation of hyperbola is

(a) $(x - 1)(y - 1) = 2$ (b) $y(x - 1) = x + 1$

(c) $x(y - 1) = y + 1$ (d) all of these

19. The number of real tangent that can be drawn from point of intersection of at which they are at right angle.

(a) 0 (b) 2 (c) 3 (d) 4

20. The equation of conjugate hyperbola is

(a) $(x - 1)(y - 1) = 4$ (b) $(x - 1)(y - 1) + 1 = 0$

(c) $(x - 1)(y - 1) = -2$ (d) none of these

Numerical Value Type

21. The sum of distances of any point on the ellipse $5x^2 + 9y^2 = 45$ from its directrix is d, then sum of the divisors of d is

22. Let the normals are drawn from (a, b) to the hyperbola $xy = 4$ and (x_i, y_i), $i = 1, 2, 3, 4$ be the feet of the co-normal points. If the algebraic sum of the perpendicular distances drawn from (x_i, y_i), $i = 1, 2, 3, 4$ onto a variable line vanishes then the variable line passing through the point $\left(\frac{a}{\lambda}, \frac{b}{\lambda}\right)$, then number of even prime factors of λ is

23. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$ at the ends of latus rectum. If the area of quadrilateral be α sq. units, then the value of α must be

24. The equation of the parabola whose focus is origin and tangent at the vertex is $x - y + 2 = 0$ is of the form $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, then the value of $|g| + |f| + |c|$ must be

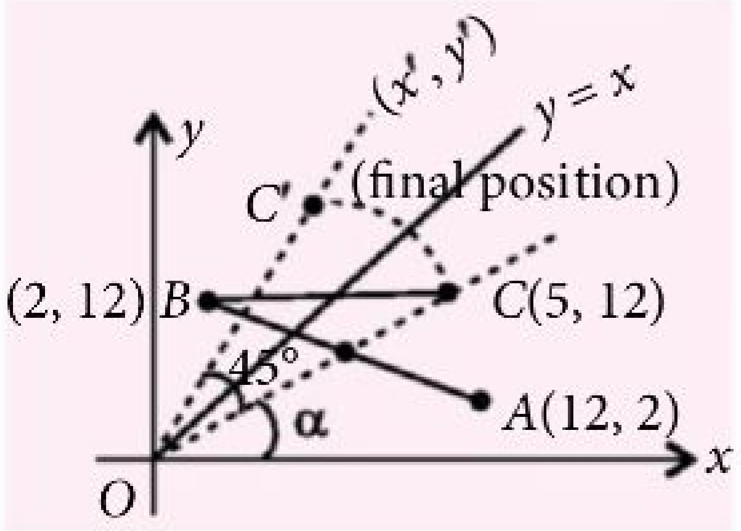
25. The equation of a line which is parallel to the line common to the pair of lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and at a distance of 7 units from it is $ax + by + c = 0$, then $|c|$ must be

26. If e_1 and e_2 are the eccentricities of hyperbolas $xy = c^2$ and $x^2 - y^2 = b^2$, then $e_1 \times e_2$ must be

SOLUTIONS

1. (b) : Given point say, $A(12, 2)$ and let the final position is (x', y').

$\therefore$ Image of point A about the line $y = x$ is $B(2, 12)$. Due to translation through 3 units along the positive direction of x-axis, the new point is $C(5, 12)$.



$$\because \quad OC = OC' = \sqrt{5^2 + 12^2} = 13$$

Thus, $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$

$\therefore$ $x' = OC' \cos(45° + \alpha)$, $y' = OC' \sin(45° + \alpha)$

$\therefore$ $(x', y') = (13(\cos 45° \cos\alpha - \sin 45° \sin\alpha),$
$13(\sin 45° \cos\alpha + \cos 45° \sin\alpha))$

$$= \left(\frac{13}{\sqrt{2}}(\cos\alpha - \sin\alpha), \frac{13}{\sqrt{2}}(\cos\alpha + \sin\alpha)\right)$$

$$= \left(\frac{13}{\sqrt{2}}\left(-\frac{7}{13}\right), \frac{13}{\sqrt{2}}\left(\frac{17}{13}\right)\right) = \left(\frac{-7}{\sqrt{2}}, \frac{17}{\sqrt{2}}\right)$$

2. (a) : Given, $x\cos\theta + y\sin\theta + \sin\theta\tan\theta = 0$ and the points are $(m^2, 2m)$, $(mm', m + m')$ and $(m'^2, 2m')$

$$\therefore \quad P_1 = \left|\frac{m^2\cos\theta + 2m\sin\theta + \sin\theta\tan\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$$

$$= \frac{|m^2\cos^2\theta + 2m\sin\theta\cos\theta + \sin^2\theta|}{|\cos\theta|} = \frac{(m\cos\theta + \sin\theta)^2}{|\cos\theta|}$$

$$P_2 = \left|mm'\cos\theta + (m + m')\sin\theta + \frac{\sin^2\theta}{\cos\theta}\right|$$

$$= \frac{|mm'\cos^2\theta + (m + m')\sin\theta\cos\theta + \sin^2\theta|}{|\cos\theta|}$$

$$= \frac{|m\cos\theta + \sin\theta||m'\cos\theta + \sin\theta|}{|\cos\theta|}$$

$$P_3 = \left| m'^2 \cos\theta + 2m' \sin\theta + \frac{\sin^2\theta}{\cos\theta} \right|$$

$$= \frac{|m'^2 \cos^2\theta + 2m' \sin\theta\cos\theta + \sin^2\theta|}{|\cos\theta|}$$

$$= \frac{(m'\cos\theta + \sin\theta)^2}{|\cos\theta|}$$

$\therefore$ $P_1P_3 = P_2^2$ $\Rightarrow$ P_1, P_2, P_3 are in G.P.

3. (c) : As $y = (3+x)^{\cos x} - \frac{2}{3}\sin x$ ...(i)

and $12y + x - 36 = 0$ ...(ii)

The point of intersection of curve and line is $(0, 3) = (\alpha, \beta)$.

Now, $y = (3+x)^{\cos x} - \frac{2}{3}\sin x$

$$\Rightarrow \quad y' = (3+x)^{\cos x}\left\{\frac{\cos x}{3+x} - \sin x \log(3+x)\right\} - \frac{2}{3}\cos x$$

$$\therefore \quad y'(0) = 3\left(\frac{1}{3} - 0\right) - \frac{2}{3} = \frac{1}{3} \text{ (say } m_1)$$

and $x + 12y - 36 = 0$ gives $y'(0) = -\frac{1}{12}$ (say m_2)

$$\text{Now, } \tan\theta = \frac{m_1 - m_2}{1 + m_1m_2} = \frac{\frac{1}{3} + \frac{1}{12}}{1 - \frac{1}{36}} = \frac{3}{7}$$

4. (b) : Let $x = t$ be any point on $2x - 3y - 5 = 0$.

Then, $y = \frac{2t-5}{3}$

Let (x, y) be the image of the point $\left(t, \frac{2t-5}{3}\right)$ in the line $x + 2y + 3 = 0$.

$$\therefore \quad \frac{x-t}{1} = \frac{y - \left(\frac{2t-5}{3}\right)}{2} = \frac{-2\left(t + 2\cdot\frac{(2t-5)}{3} + 3\right)}{5}$$

$$\Rightarrow \quad \frac{x-t}{1} = \frac{3y - (2t-5)}{6} = -\frac{2}{15}(7t-1)$$

$\Rightarrow$ $15(x - t) = -14t + 2$ and $5(3y + 5 - 2t) = -4(7t - 1)$

$\Rightarrow$ $15x - 2 = t$ and $-\frac{(5y+7)}{6} = t$

$$\Rightarrow \quad 15x - 2 = -\frac{(5y+7)}{6} \quad \Rightarrow 18x + y - 1 = 0$$

5. (c) : As origin is shifted to $(2, -3)$

$\therefore$ $x \to x + 2$, $y \to y - 3$ and the equation is transformed to $(y - 3)^2 + 8(x + 2) + 6(y - 3) - 7 = 0$

$\Rightarrow$ $y^2 + 8x + 9 - 6y + 16 + 6y - 18 - 7 = 0$

$\Rightarrow$ $y^2 = -8x = 4(-2)x = 4(a)x$

$\therefore$ $a = -2$

6. (d) : We have, $(x - 4)^2 + (y - 5)^2 = m^2$ ...(i)

and $5x + 4y - 40 = 0$ or $\frac{5x+4y}{40} = 1$ ...(ii)

From (i), we have

$x^2 + y^2 - 8x - 10y + (41 - m^2) = 0$ ...(iii)

Homogenizing (i) by the line (ii), we have

$$x^2 + y^2 - (8x + 10y)\left(\frac{5x+4y}{40}\right) + (41 - m^2)\left(\frac{5x+4y}{40}\right)^2 = 0$$

The above equation will represent the pair of perpendicular lines if coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad \left(1 - 1 + \frac{41-m^2}{64}\right) + \left(1 - 1 + \frac{41-m^2}{100}\right) = 0$$

$\Rightarrow$ $100(41 - m^2) + 64(41 - m^2) = 0$

$\Rightarrow$ $164(41 - m^2) = 0 \Rightarrow m^2 = 41$.

7. (a) : We know that the chord of $xy = c^2$ whose midpoint is (h, k) is $\frac{x}{h} + \frac{y}{k} = 2$ ...(i)

Also, $(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Now, by (i), we have

$$\frac{x}{\frac{x_1+x_2}{2}} + \frac{y}{\frac{y_1+y_2}{2}} = 2 \Rightarrow \frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$$

8. (b) : Given, $xdy + ydx = 0 \Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$

$\Rightarrow$ $\log(xy) = \log e$ $\Rightarrow$ $xy = e$

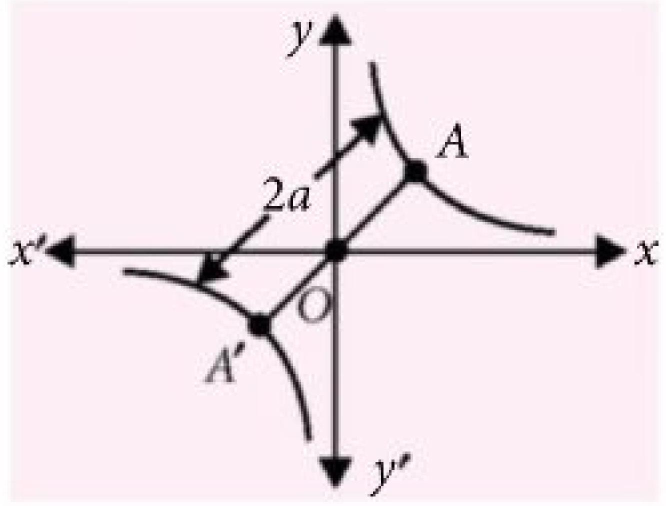
$\Rightarrow$ $xy = 36$ ...(i) [$\because$ conic passes through $(4, 9)$]

$\therefore$ Conic is a rectangular hyperbola.

$\therefore$ Eccentricity $(e) = \sqrt{2}$ and length of latus rectum $= (2b^2/a) = 2a$ ($\because a = b$)

Now, the transverse axis is along the line $y = x$ ...(ii)

Solving (i) and (ii), we have



$x^2 = 36$

$\Rightarrow$ $x = \pm 6$

Then, $y = \pm 6$

$\therefore$ Vertices are $A(6, 6)$ and $A'(-6, -6)$

$\therefore$ Length of latus rectum

$$= AA' = 2a = \sqrt{144 \times 2} = 12\sqrt{2}$$

9. (d) : The equations of asymptotes are

$3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$

The combined equation of asymptotes is

$(3x - 4y + 7)(4x + 3y + 1) = 0$

or $12x^2 - 7xy - 12y^2 + 31x + 17y + 7 = 0$ ...(i)

Let the equation of hyperbola be

$12x^2 - 7xy - 12y^2 + 31x + 17y + \lambda = 0$ ...(ii)

Since, (i) passes through (0, 1) $\therefore$ $\lambda = -5$

$\therefore$ Equation of hyperbola is

$12x^2 - 7xy - 12y^2 + 31x + 17y - 5 = 0$

Equation of conjugate hyperbola = 2(combined equation of asymptotes) – equation of hyperbola

$\Rightarrow$ Equation of conjugate hyperbola

$= 2(12x^2 - 7xy - 12y^2 + 31x + 17y + 7)$

$- (12x^2 - 7xy - 12y^2 + 31x + 17y - 5) = 0$

$\Rightarrow$ $12x^2 - 7xy - 12y^2 + 31x + 17y + 19 = 0$ ...(iii)

Comparing (iii) with $12x^2 - 7xy - 12y^2 + ax + by + c = 0$, we have $a = 31$, $b = 17$, $c = 19$

10. (b) : Let e and e' be the eccentricities of hyperbola and its conjugate.

Then $\frac{1}{e^2} + \frac{1}{e'^2} = 1 \Rightarrow \frac{1}{(e')^2} = 1 - \frac{1}{e^2}$ ...(i)

Since, we know that the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by $2\tan^{-1}(b/a)$

$\therefore$ $2\tan^{-1}(b/a) = \pi/6 \Rightarrow \frac{b}{a} = \tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$

$$\Rightarrow \frac{b^2}{a^2} = (2 - \sqrt{3})^2 = \frac{7 - 4\sqrt{3}}{1}$$

$$\Rightarrow \frac{b^2 + a^2}{a^2} = \frac{8 - 4\sqrt{3}}{1} = e^2 \Rightarrow \frac{1}{e^2} = \frac{8 + 4\sqrt{3}}{16}$$

$$\therefore \text{ By (i), } \frac{1}{(e')^2} = 1 - \frac{8 + 4\sqrt{3}}{16} = \frac{8 - 4\sqrt{3}}{16} = \frac{2 - \sqrt{3}}{4}$$

$$\Rightarrow (e')^2 = \frac{2(4 + 2\sqrt{3})}{1} = (\sqrt{2})^2(1 + \sqrt{3})^2$$

$$\Rightarrow e' = \sqrt{2} + \sqrt{6}$$

11. (a, b, c, d) : Let equation of asymptotes of hyperbola $3x^2 + 10xy + 8y^2 + 14x + 22y + 7 = 0$ is given by

$3x^2 + 10xy + 8y^2 + 14x + 22y + \lambda = 0$ ...(i)

Now, (i) is combined equation of asymptotes *i.e.*, (i) represents pair of straight line.

Then, $\Delta = 0$ *i.e.*, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow$ $3 \cdot 8 \cdot \lambda + 2(7)(11)(5) - 3(121) - 8(49) - 25\lambda = 0$

$\Rightarrow$ $\lambda = 15$

$\therefore$ Combined equation of asymptotes is

$3x^2 + (10y + 14)x + (8y^2 + 22y + 15) = 0$

$$\therefore x = \frac{-(10y + 14) \pm \sqrt{4y^2 + 16y + 16}}{6}$$

$\Rightarrow$ $3x = -(5y + 7) \pm (y + 2)$

$\therefore$ Equation of asymptotes are $3x + 4y + 5 = 0$ and $x + 2y + 3 = 0$ (option (b))

Now, the point of intersection of these asymptotes is the centre of hyperbola *i.e.*, $x = 1$, $y = -2$ (option (a))

$\therefore$ Centre is $(1, -2)$

Also, equation of conjugate hyperbola = 2(combined equation of asymptotes) – (equation of given hyperbola)

$\therefore$ Conjugate hyperbola is $2(3x^2 + 10xy + 8y^2 + 14x + 22y + 15) - (3x^2 + 10xy + 8y^2 + 14x + 22y + 7) = 0$

$\Rightarrow$ $3x^2 + 10xy + 8y^2 + 14x + 22y + 23 = 0$

Again, angle between the asymptotes (θ), is given by

$$\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a + b}\right) = \tan^{-1}\left(\frac{2}{11}\right)$$

12. (a, b, c, d)

13. (b, c) : Here, equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Now, equation of tangent at $P(\theta) = P(4\cos\theta, 3\sin\theta)$ is given by $3x\cos\theta + 4y\sin\theta = 12$ ...(i)

Now, (i) passes through the centre (4, 3).

COMIC CAPSULE

"I found this in the hall. Call Accounting and find out if they've misplaced a decimal point again."

$\therefore \quad 12\cos\theta + 12\sin\theta = 12$

$\Rightarrow \quad \cos\theta + \sin\theta = 1 \quad \Rightarrow \quad \theta = 0 \text{ or } \pi/2$

14. (a, b, c, d) : Equation of chord of ellipse with $\left(\frac{1}{2}, \frac{2}{5}\right)$ as its mid-point is $\frac{1}{4\times25}+\frac{4}{25\times16}-1=\frac{\frac{1}{2}x}{25}+\frac{\frac{2}{5}y}{16}-1$

(Using $T = S_1$)

$\Rightarrow \quad 4x + 5y = 4$ ...(i)

Now, solving (i) and ellipse, we get

$16x^2 + 16(1 - x)^2 = 400$ $[\because 5y = 4(1 - x)]$

$\Rightarrow \quad x^2 - x - 12 = 0 \quad \therefore \; x' \text{ and } x'' = 4, -3$

When, $x' = 4$, then $y' = \frac{-12}{5}$

and when $x'' = -3$, then $y'' = \frac{16}{5}$

Again, length of chord $AB = \sqrt{7^2+\left(\frac{28}{5}\right)^2} = \frac{7\sqrt{41}}{5}$

15. (a, b, c, d) : We have, $x^2 - 8x + 4y + \lambda = 0$

$\Rightarrow \quad (x-4)^2 = -4\left(y+\frac{\lambda-16}{4}\right)$ ...(i)

$\Rightarrow \quad X^2 = 4(-1)Y$, where $X = x - 4$, $Y = \left(y+\frac{\lambda-16}{4}\right)$

$\therefore$ Equation of directrix is $Y = 1$

i.e., $y+\frac{\lambda-16}{4}=1 \Rightarrow \quad y+2=3+\frac{16-\lambda}{4}=0 \quad \Rightarrow \lambda=28$

$\therefore$ Equation (i) reduces to $(x - 4)^2 = -4(y + 3)$

Thus, vertex $(4, -3)$ and focus, $X = 0$ and $Y = -1$ *i.e.*, $(4, -4)$.

16. (a, b, c, d) : The given two lines will be identical if there exist a real value k such that

$a^3 - b^3 = k(a - b)$, $b^3 - c^3 = k(b - c)$ and $c^3 - a^3 = k(c - a)$

$\Rightarrow \quad a - b = 0$ or $a^2 + ab + b^2 = k$ and so for others.

$\therefore \quad a = b$ or $b = c$ or $c = a$

Again, $a^2 + ab + b^2 = b^2 + c^2 + bc = k$

$\Rightarrow \quad a^2 - c^2 = b(c - a) \quad \Rightarrow \quad (a + c)(a - c) = -b(a - c)$

$\Rightarrow \quad a + b + c = 0$ or $a = c$

17. (b) : Since, the point of intersection of perpendicular tangents is $(1, 1)$.

$\therefore$ $(1, 1)$ is centre of the rectangular hyperbola.

$\therefore$ Equation of asymptotes are $(x - 1) = 0$ and $(y - 1) = 0$.

So the joint equation of asymptotes is $(x - 1)(y - 1) = 0$.

18. (d) : We know that joint equation of asymptotes and hyperbola differ by a constant only.

Let equation of hyperbola is $(x - 1)(y - 1) + \lambda = 0$

Since it passes through the orthocentre $(3, 2)$

$\therefore \quad \lambda + 2 = 0$

$\therefore$ Equation of hyperbola is $(x - 1)(y - 1) = 2$.

19. (b) : From the centre of hyperbola, two real tangents can be drawn to the hyperbola.

20. (c) : We know that hyperbola conjugate to $xy = c^2$ is $xy = -c^2$.

$\therefore$ Equation of hyperbola conjugate to $(x - 1)(y - 1) = 2$ is $(x - 1)(y - 1) = -2$.

21. (13) : The given equation of ellipse is $5x^2 + 9y^2 = 45$

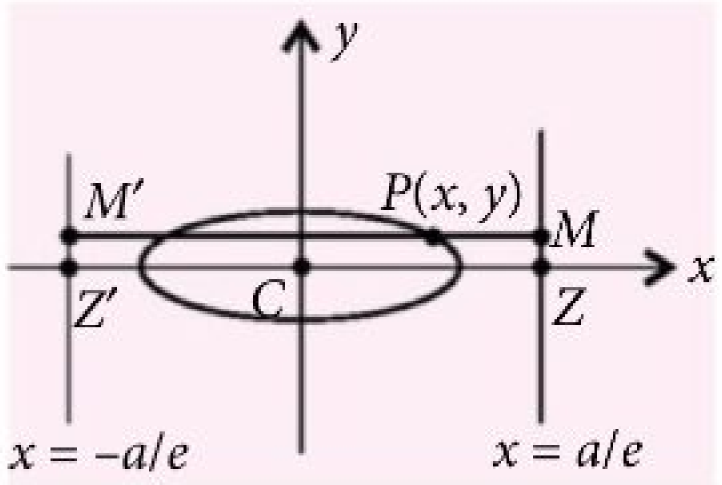
$\Rightarrow \quad \frac{x^2}{9}+\frac{y^2}{5}=1 \Rightarrow a=3, b=\sqrt{5}, e=\frac{2}{3}$

$\therefore$ Sum of distances of point $P(x, y)$ from its directrices

$= PM + PM' = MM'$

$= ZZ' = 2CZ$

$=\frac{2a}{e}=9=d$



$\therefore$ Sum of divisors of d is $\frac{3^3-1}{3-1}=13$

22. (1) : Given, $xy = 4 = 2^2$.

On comparing with $xy = c^2$, we get $c = 2$

Let $\left(2t, \frac{2}{t}\right)$ be any point on the hyperbola $xy = 4$

Now equation of normal to the hyperbola $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ is $xt^3 - yt = c(t^4 - 1)$.

$\Rightarrow \quad xt^3 - yt = 2(t^4 - 1)$ $(\because c = 2)$

$\Rightarrow \quad 2t^4 - xt^3 + yt - 2 = 0$ ...(i)

which passes through (a, b)

$\therefore \quad 2t^4 - at^3 + bt - 2 = 0$

If foot of the co-normal points are $(t_i, 1/t_i)$, $i = 1, 2, 3, 4$, then $(t_1 + t_2 + t_3 + t_4) = a/2$

$\Rightarrow \quad x_1+x_2+x_3+x_4=\frac{a}{2}$

and $\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\frac{1}{t_4}\right)=\frac{b}{2}$ or $y_1+y_2+y_3+y_4=\frac{b}{2}$

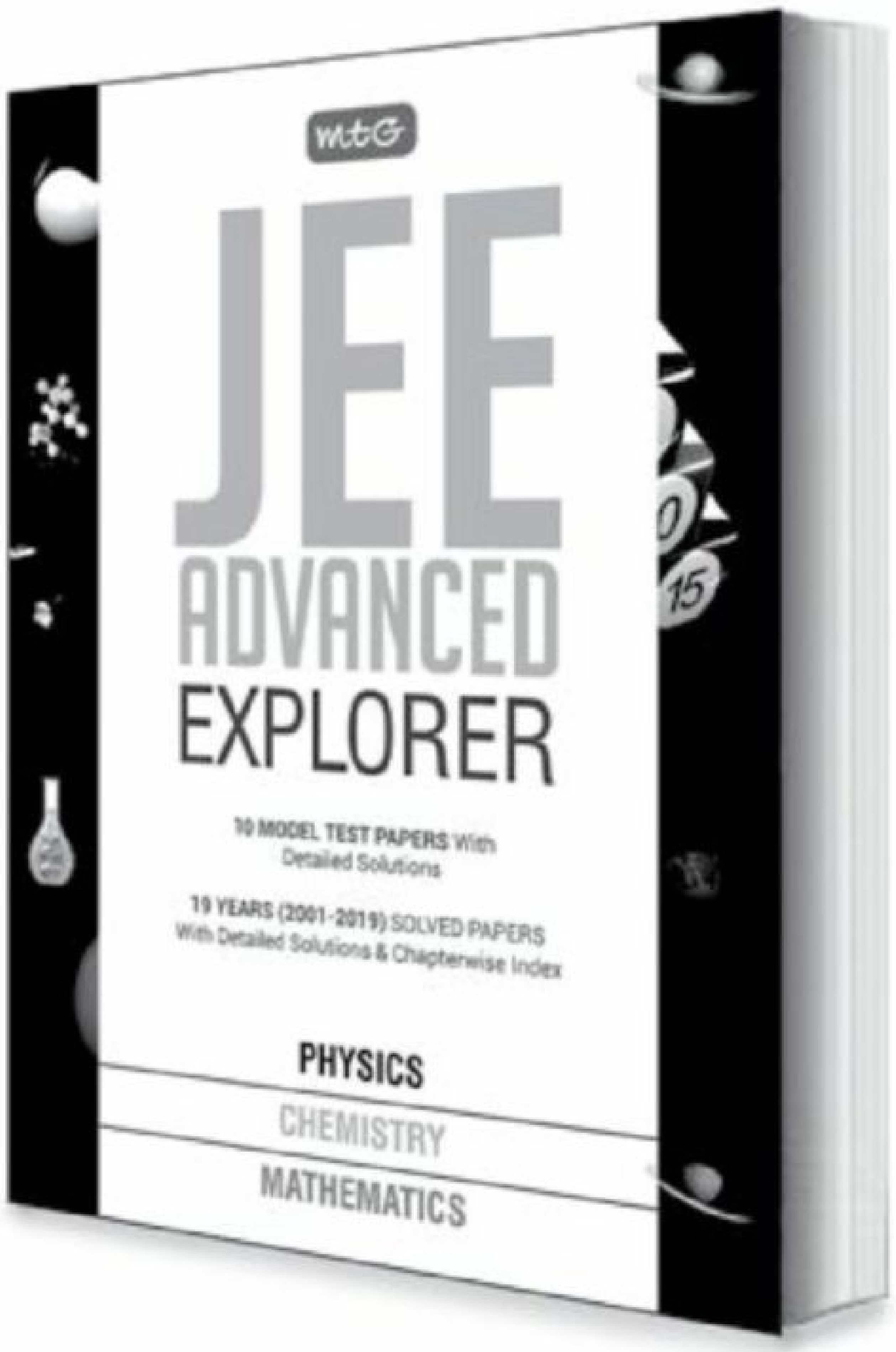
Let the variable line be $lx + my + n = 0$ ...(ii)

Since the algebraic sum of $\perp$ distance from (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) to equation (ii) is vanishes.

$\Rightarrow \quad l(x_1 + x_2 + x_3 + x_4) + m(y_1 + y_2 + y_3 + y_4) + 4n = 0$

Precision Revision for JEE is Here!

MtG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$$\Rightarrow\quad l\left(\frac{x_1+x_2+x_3+x_4}{4}\right)+m\left(\frac{y_1+y_2+y_3+y_4}{4}\right)+n=0$$

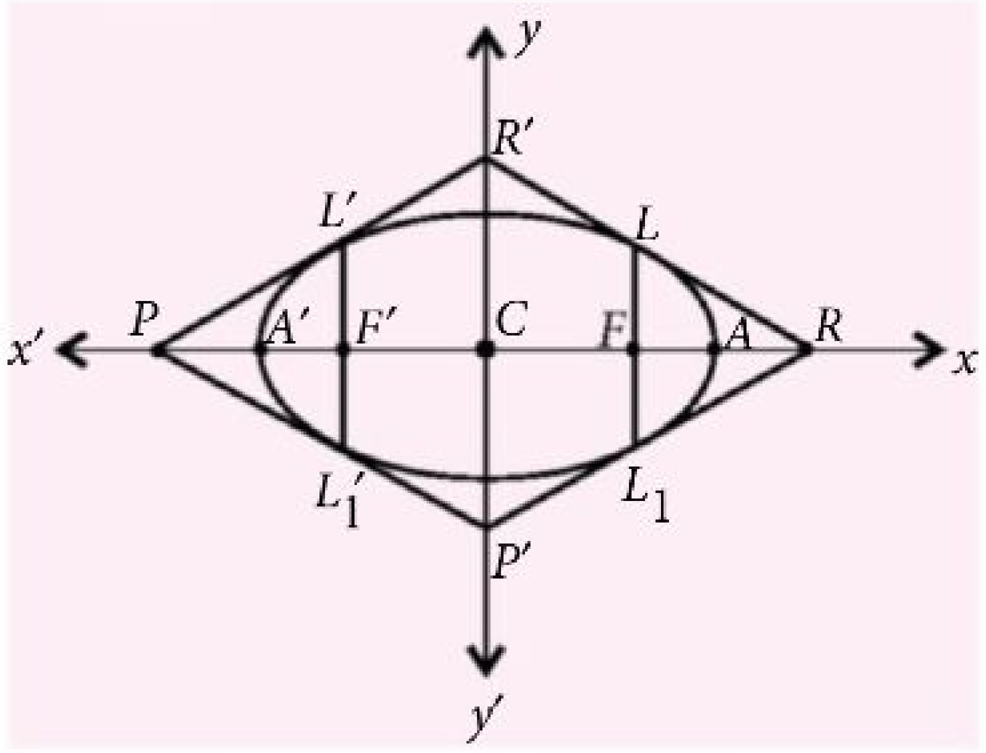
$$\Rightarrow\quad l\left(\frac{a}{8}\right)+m\left(\frac{b}{8}\right)+n=0$$

$\Rightarrow$ Equation (ii) passing through $\left(\frac{a}{8}, \frac{b}{8}\right)$

$\therefore$ $\lambda = 8 = 2^3 \Rightarrow 2$ is only the even prime factor of λ which repeated thrice.

$\therefore$ Number of prime factor of λ which are even is only 1.

23. (54) :



The given equation of ellipse is $\frac{x^2}{9}+\frac{y^2}{8}=1$

$$\Rightarrow\quad a^2=9,\ b^2=8\ \Rightarrow\ e=\frac{1}{3}\ \text{(eccentricity)}$$

Now, $L\left(ae, \frac{b^2}{a}\right)=L\left(1, \frac{8}{3}\right)$

$\therefore$ Equation of tangent at $\left(1, \frac{8}{3}\right)$ is $\frac{x}{9}+\frac{y}{3}=1$ which meets x axis and y axis at $R(9, 0)$ and $R'(0, 3)$.

$\therefore$ $CR = 9$ and $CR' = 3$

Now, area of $\Delta\ CRR'=\frac{1}{2}CR\times CR'=\frac{1}{2}\times 9\times 3=\frac{27}{2}$

$\therefore$ Area of quadrilateral $PP'RR' = 4 \times$ Area of $\Delta CRR'$

$$=4\times\frac{27}{2}=54\text{ sq.units}\quad\therefore\ \alpha = 54$$

24. (24) : Given, focus of parabola is (0, 0) and equation of tangent at vertex is $x - y + 2 = 0$. ...(i)

Now, axis of parabola is the line through focus and perpendicular to the directrix but focus and vertex lies on the same line.

$\therefore$ Equation of axis of parabola is $x + y = 0$ ...(ii)

The point of intersection of axis and tangent line is obtained by solving (i) and (ii), which is the vertex $= (-1, 1)$

Let equation of directrix is $x - y + \lambda = 0$ ...(iii)

Since, (iii) passes through $(-2, 2)$ so $\lambda = 4$.

$\therefore$ (iii) becomes $x - y + 4 = 0$

Now, using the definition of parabola, we have

$$x^2+y^2=\left(\frac{x-y+4}{\sqrt{2}}\right)^2\ \Rightarrow\ 2(x^2+y^2)=(x-y+4)^2$$

$$\Rightarrow\ x^2+y^2+2xy-8x+8y-16=0$$
$$=ax^2+by^2+2hxy+2gx+2fy+c$$

$$\Rightarrow\ |g|+|f|+|c|=|-4|+|4|+|-16|=24$$

25. (35) : Given, $6x^2 - xy - 12y^2 = 0$

$\Rightarrow$ $(3x + 4y)(2x - 3y) = 0$ ...(i)

Also, $15x^2 + 14xy - 8y^2 = 0$

$\Rightarrow$ $(3x + 4y)(5x - 2y) = 0$ ...(ii)

$\therefore$ Line common to (i) and (ii) is $3x + 4y = 0$ ...(iii)

Now, any line parallel to (iii) is $3x + 4y + \lambda = 0$ which is at a distance of 7 units from (iii). $\therefore$ $\lambda = \pm 35$

Hence, $|c| = 35$

26. (2) : Since, both $xy = c^2$ and $x^2 - y^2 = b^2$ are rectangular hyperbola.

$$\therefore\ e_1=\sqrt{2}=e_2\ \Rightarrow\ e_1\times e_2=2$$

◆◆

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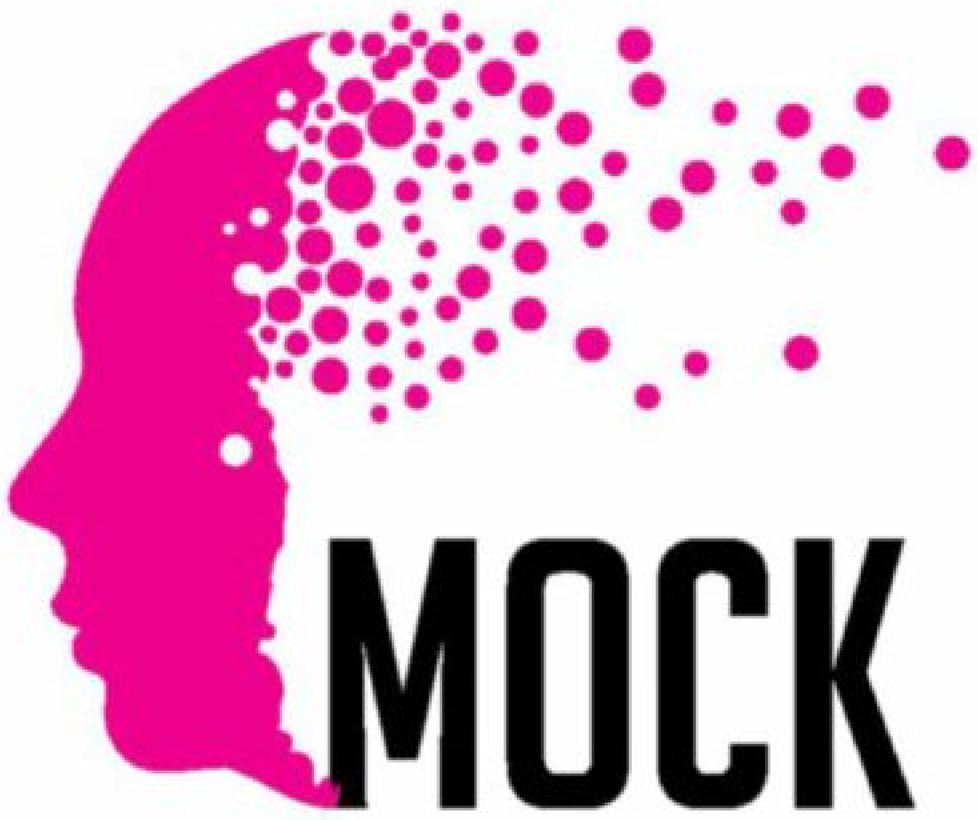

Exam Date :

Between 6th to 11th January

MOCK DRILL FOR JEE MAIN 2020

***ALOK KUMAR**

1. In a triangle ABC, with $A=\frac{\pi}{7}, B=\frac{2\pi}{7}, C=\frac{4\pi}{7}$, then $a^2+b^2+c^2$ is (R = circumradius of ΔABC)

(a) $4R^2$ (b) $6R^2$ (c) $7R^2$ (d) $8R^2$

2. Range of $f(x)=\sin^6 x+\cos^6 x$ is

(a) $[0, 1]$ (b) $[0, \sqrt{2}]$

(c) $\left[\frac{1}{\sqrt{2}}, \frac{3}{4}\right]$ (d) $\left[\frac{1}{4}, 1\right]$

3. Let $f(x)=x^2+5x+6$, then the number of real roots of $(f(x))^2+5f(x)+6-x=0$ is

(a) 1 (b) 2 (c) 3 (d) 0

4. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

(a) 0 (b) 1 (c) 2 (d) infinite

5. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2+z_2^2+z_3^2+z_4^2$ equals

(a) $z_1z_2+z_2z_3+z_3z_4+z_4z_1$

(b) $z_1z_2+z_1z_3+z_1z_4+z_2z_3+z_2z_4+z_3z_4$

(c) 0 (d) none of these

6. If $|z-3i|=3$, (where $i=\sqrt{-1}$) and $\arg z \in (0, \pi/2)$, then $\cot(\arg(z))-6/z$ is equal to

(a) 0 (b) $-i$

(c) i (d) none of these

7. If the arithmetic mean of two positive numbers a and $b (a > b)$ is twice their G.M., then $a : b$ is

(a) $6+\sqrt{7}:6-\sqrt{7}$ (b) $2+\sqrt{3}:2-\sqrt{3}$

(c) $5+\sqrt{6}:5-\sqrt{6}$ (d) none of these

8. For each positive integer n, let

$$s_n=\frac{3}{1\cdot 2\cdot 4}+\frac{4}{2\cdot 3\cdot 5}+\frac{5}{3\cdot 4\cdot 6}+\ldots\ldots+\frac{n+2}{n(n+1)(n+3)}.$$

Then $\lim\limits_{n\to\infty} s_n$ equals

(a) $\frac{29}{6}$ (b) $\frac{29}{36}$ (c) 0 (d) $\frac{29}{18}$

9. $\lim\limits_{x\to\infty} \sqrt[3]{x}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)=$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

10. The value of $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ is

(a) 0 (b) 1/2 (c) 1/4 (d) 1

11. The range of $\sin^{-1}x-\cos^{-1}x$ is

(a) $\left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[\frac{-5\pi}{2}, \frac{\pi}{3}\right]$

(c) $\left[\frac{-3\pi}{2}, \pi\right]$ (d) $\left[0, \frac{\pi}{2}\right]$

12. A real valued function f satisfies $f(10+x)=f(10-x)$ and $f(20-x)=-f(20+x)$, for all $x \in R$. Which of the following statements is true?

(a) f is an even function

(b) f is an odd function

(c) f is a constant function

(d) f is a non-periodic function

13. Number of pairs of positive integers (p, q) whose L.C.M. (Least common multiple) is 8100, is "K". Find the number of ways of expressing K as a product of two coprime numbers.

(a) 2 (b) 6 (c) 4 (d) 8

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*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

14. If $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(a) $4B$ (b) $128B$ (c) $-128B$ (d) $-64B$

15. In ΔABC, if $A - B = 120°$ and $R = 8r$, then the value of $\dfrac{1+\cos C}{1-\cos C}$ equals (All symbols used have their usual meaning in a triangle)

(a) 12 (b) 15 (c) 21 (d) 31

16. The domain of the derivative of the function

$$f(x)=\begin{cases} \tan^{-1} x, \text{ if } |x|\le 1 \\ \dfrac{1}{2}(|x|-1), \text{ if } |x|>1 \end{cases} \text{ is}$$

(a) $R - \{0\}$ (b) $R - \{1\}$
(c) $R - \{-1\}$ (d) $R - \{-1, 1\}$

17. In ΔABC orthocentre is (6,10) circumcentre is (2, 3) and equation of side $\overrightarrow{BC}$ is $2x + y = 17$. Then the radius of the circumcircle of ΔABC is

(a) 4 (b) 5 (c) 2 (d) 3

18. An equilateral triangle has its centroid at origin and one side is $x + y = 1$. The equations of the other sides are

(a) $y+1=(2\pm\sqrt{3})(x+1)$
(b) $y+1=(2\pm\sqrt{3})x,\ y+1=(3\pm\sqrt{3})x$
(c) $y+1=(3\pm\sqrt{3})(x-1),\ y+1=\sqrt{3}x$
(d) $y\pm 1=(3\pm\sqrt{3})(x-1),\ y+1=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}(x+1)$

19. From origin, chords are drawn to the circle $x^2 + y^2 - 2y = 0$. The locus of the middle points of these chords is

(a) $x^2 + y^2 - y = 0$ (b) $x^2 + y^2 - x = 0$
(c) $x^2 + y^2 - 2x = 0$ (d) $x^2 + y^2 - x - y = 0$

20. The locus of the mid point of the chord of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$ which makes an angle of 120°at the centre is

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$
(b) $x^2 + y^2 + x + y + 1 = 0$
(c) $x^2 + y^2 - 2x - 2y - 1 = 0$
(d) $x^2 + y^2 + x - y - 1 = 0$

Numerical Value Type

21. For non negative integers m, n define a function as follows

$$f(m,n)=\begin{cases} n+1 & , \text{ if } \quad m=0 \\ f(m-1,1) & , \text{ if } \quad m\ne 0, n=0 \\ f(m-1, f(m,n-1)), & \text{ if } \quad m\ne 0, n\ne 0 \end{cases}$$

Then the value of $f(1, 1)$ is

22. If $\log_{0.5}\sin x = 1 - \log_{0.5}\cos x$, then the number of values of $x \in [-2\pi, 2\pi]$ is

23. If the function $f(x)=\dfrac{x-1}{c-x^2+1}$ does not take any value in the interval $\left[-1,-\dfrac{1}{3}\right]$, then number of positive integral values of c is

24. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then absolute value of $\alpha^{15} + \beta^{15}$ equals

25. The number of solutions of the equation $\sin^5 x-\cos^5 x=\dfrac{1}{\cos x}-\dfrac{1}{\sin x}(\sin x\ne\cos x)$ is

SOLUTIONS

1. (c) : $a^2 + b^2 + c^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$

$$= 2R^2(1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C)$$

$$=2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}\right)\right]$$

$$=2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)\right]$$

$$=2R^2\left[3-\left(2\sin\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{6\pi}{7}\right)\frac{1}{2\sin\frac{\pi}{7}}\right]$$

$$=2R^2\left[3-\frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\frac{7\pi}{7}-\sin\frac{5\pi}{7}\right)\right]$$

$$=2R^2\left[3+\frac{1}{2}\right]=7R^2$$

2. (d) : $f(x) = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x\,(\sin^2 x + \cos^2 x)$

$$=1-\frac{3}{4}\sin^2 2x$$

Range of $\sin^2 2x$ is [0, 1].

$\therefore$ Range of $f(x)$ is $\left[\dfrac{1}{4},1\right]$.

3. (d) : Use "$f(x) = x$ has non real roots $\Rightarrow f(f(x)) = x$ also has non-real roots"

4. (d) : Given, $f(x) = x - [x]$, $x \in R - \{0\}$

Now, $f(x)+f\left(\dfrac{1}{x}\right)=1$

$$\therefore\ x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1 \Rightarrow \left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$$

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$$\Rightarrow \quad \left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1 \qquad \ldots\text{(i)}$$

Clearly, R.H.S. is an integer L.H.S. is also an integer

Let $x+\frac{1}{x}=k$ (an integer) $\Rightarrow$ $x^2-kx+1=0$

$$\therefore \quad x=\frac{k\pm\sqrt{k^2-4}}{2}$$

For real values of x, $k^2 - 4 \geq 0 \Rightarrow k \geq 2$ or $k \leq -2$

We also observe that $k = 2$ and -2 does not satisfy equation (i).

$\therefore$ The equation (i) will have solutions if $k > 2$ or $k < -2$, where $k \in Z$.

Hence, equation (i) has infinite number of solutions.

5. (a) : We know that, $\frac{z_2-z_1}{z_4-z_1}=\frac{|z_2-z_1|}{|z_4-z_1|}e^{i\pi/2}=i$

(as $|z_2 - z_1| = |z_4 - z_1|$)

$\Rightarrow$ $(z_4 - z_1)^2 + (z_2 - z_1)^2 = 0$ ....(i)

Similarly, $\frac{z_4-z_3}{z_2-z_3}=-i$

$\Rightarrow$ $(z_4 - z_3)^2 + (z_2 - z_3)^2 = 0$ ....(ii)

On adding (i) and (ii) , we get

$$2[z_1^2+z_2^2+z_3^2+z_4^2-z_1z_2-z_4z_1-z_4z_3-z_2z_3]=0$$

$$\Rightarrow \quad z_1^2+z_2^2+z_3^2+z_4^2=z_1z_2+z_2z_3+z_3z_4+z_4z_1$$

6. (c)

7. (b) : Given, $\frac{a+b}{2}=2\sqrt{ab} \Rightarrow \quad a+b-4\sqrt{ab}=0$

$\Rightarrow$ $\frac{a}{b}+1-4\sqrt{\frac{a}{b}}=0$ (Dividing by b)

$$\Rightarrow \quad \left(\sqrt{\frac{a}{b}}\right)^2-4\sqrt{\frac{a}{b}}+1=0$$

$$\therefore \quad \sqrt{\frac{a}{b}}=\frac{4\pm2\sqrt{3}}{2}=\left(2\pm\sqrt{3}\right) \quad \Rightarrow \quad \frac{a}{b}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

8. (b) : Let $u_k=\frac{k+2}{k(k+1)(k+3)}=\frac{(k+2)^2}{k(k+1)(k+2)(k+3)}$

$$=\frac{k^2+4k+4}{k(k+1)(k+2)(k+3)}=\frac{k(k+1)+3k+4}{k(k+1)(k+2)(k+3)}$$

$$=\frac{1}{(k+2)(k+3)}+\frac{3}{(k+1)(k+2)(k+3)}+\frac{4}{k(k+1)(k+2)(k+3)}$$

$$=\left(\frac{1}{k+2}-\frac{1}{k+3}\right)-\frac{3}{2}\left[\frac{1}{(k+2)(k+3)}-\frac{1}{(k+1)(k+2)}\right]$$
$$-\frac{4}{3}\left[\frac{1}{(k+1)(k+2)(k+3)}-\frac{1}{k(k+1)(k+2)}\right]$$

Now, put $k = 1, 2, 3, \ldots.., n$ and add. Thus

$s_n = u_1 + u_2 + \ldots. + u_n$

$$=\left(\frac{1}{3}-\frac{1}{n+3}\right)-\frac{3}{2}\left[\frac{1}{(n+2)(n+3)}-\frac{1}{2\cdot3}\right]$$
$$-\frac{4}{3}\left[\frac{1}{(n+1)(n+2)(n+3)}-\frac{1}{1\cdot2\cdot3}\right]$$

Therefore, $\lim\limits_{n\to\infty} s_n=\frac{1}{3}+\frac{3}{12}+\frac{4}{18}=\frac{29}{36}$

9. (d) : $\lim\limits_{x\to\infty} x^{1/3}\left\{(x+1)^{1/3}+(x-1)^{1/3}\right\}$
$$\left\{(x+1)^{1/3}-(x-1)^{1/3}\right\}$$

$$=\lim_{x\to\infty}\frac{x^{1/3}\left\{(x+1)^{1/3}+(x-1)^{1/3}\right\}2}{\left\{(x+1)^{2/3}+(x^2-1)^{1/3}+(x-1)^{2/3}\right\}}\text{[Rationalising]}$$

$$=\lim_{x\to\infty}\frac{2\cdot x^{2/3}\left\{\left(1+\frac{1}{x}\right)^{1/3}+\left(1-\frac{1}{x}\right)^{1/3}\right\}}{x^{2/3}\left\{\left(1+\frac{1}{x}\right)^{2/3}+\left(1-\frac{1}{x^2}\right)^{1/3}+\left(1-\frac{1}{x}\right)^{2/3}\right\}}$$

$$=\frac{2\times2}{3}=\frac{4}{3}$$

10. (b) : $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

$$=\lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{\sqrt{x^3}}}}+1}=\frac{\sqrt{1+0}}{\sqrt{1+0+0}+1}=\frac{1}{2}$$

11. (a) : $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{2}-\cos^{-1}x-\cos^{-1}x$

$$=\frac{\pi}{2}-2\cos^{-1}x$$

Now, $0\leq\cos^{-1}x\leq\pi \;\Rightarrow\; 0\leq2\cos^{-1}x\leq2\pi$

$$\Rightarrow \; -2\pi\leq-2\cos^{-1}x\leq0 \;\Rightarrow\; \frac{-3\pi}{2}\leq\frac{\pi}{2}-2\cos^{-1}x\leq\frac{\pi}{2}$$

Monthly Test Drive-7 CLASS XI ANSWER KEY

1.	(b)	2.	(d)	3.	(b)	4.	(c)
5.	(a)	6.	(b)	7.	(a,c,d)	8.	(c)
9.	(a,b,c,d)	10.	(a,d)	11.	(a,c)	12.	(a,b,c)
13.	(a,b,c)	14.	(c)	15.	(c)	16.	(a)
17.	(27)	18.	(6)	19.	(0.25)	20.	(2)

$\therefore$ Range $= \left[\frac{-3\pi}{2}, \frac{\pi}{2}\right]$

12. (b) : Change x to $10 - x$ to obtain

$f(20 - x) = f(x)$ ...(i)

Given, $f(20 - x) = -f(20 + x)$

$\Rightarrow f(x) = -f(20 + x)$ [Using (i)] ...(ii)

Now change x to $20 + x$

$f(20 + x) = -f(40 + x)$

$\Rightarrow -f(x) = -f(40 + x)$ [Using (ii)]

$\Rightarrow f(x) = f(40 + x)$, so f is periodic

Again, $f(-x) = -f(20 - x) = -f(x)$ [Using (i)]

Thus, f is odd.

13. (a) : Given, L.C.M. $(p, q) = 2^2\, 3^4\, 5^2$

$p = 2^{a_1}\, 3^{b_1}\, 5^{c_1}$, $q = 2^{a_2}\, 3^{b_2}\, 5^{c_2}$

$\Rightarrow \max\{a_1, a_2\} = 2 \Rightarrow 5$ ways

$\Rightarrow \max\{b_1, b_2\} = 4 \Rightarrow 9$ ways

$\Rightarrow \max\{c_1, c_2\} = 2 \Rightarrow 5$ ways

$\therefore$ $K = 3^2\, 5^2$, which can be expressed as in two ways.

14. (b) : We have $A = iB$

$$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \Rightarrow A^8 = 64B^2 = 128B$

15. (b) : $\frac{r}{R} = \cos A + \cos B + \cos C - 1$

$$\Rightarrow \frac{1}{8} = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - 1 + \cos C$$

$$\Rightarrow \frac{1}{8} = \sin\frac{C}{2} - 2\sin^2\frac{C}{2} \Rightarrow \sin\frac{C}{2} = \frac{1}{4}$$

$$\therefore \cos C = 1 - \frac{1}{8} = \frac{7}{8}$$

16. (d) : The given function is

$$f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \le 1 \\ \frac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(-x-1), & \text{if } x < -1 \\ \tan^{-1} x, & \text{if } -1 \le x \le 1 \\ \frac{1}{2}(x-1), & \text{if } x > 1 \end{cases}$$

Clearly L.H.L. (at $x = -1$)

$$= \lim_{h\to 0} f(-1-h) = \lim_{h\to 0} \frac{1}{2}(1+h-1) = 0$$

R.H.L. (at $x = -1$) $= \lim_{h\to 0} f(-1+h)$

$$= \lim_{h\to 0} \tan^{-1}(-1+h) = -\frac{\pi}{4}$$

$\therefore$ L.H.L. $\ne$ R.H.L. at $x = -1$

$\therefore$ $f(x)$ is discontinuous at $x = -1$.

Also we can prove in the same way, that $f(x)$ is discontinuous at $x = 1$.

$\therefore$ $f(x)$ can not be found for $x = \pm 1$ or domain of $f'(x) = R - \{-1, 1\}$.

17. (b) : Image of orthocenter of ΔABC w.r.t. $\overrightarrow{BC}$ lies on the circle.

18. (a) : Third vertex A lies on $x - y = 0$ and in III$^{\text{rd}}$ quadrant.

Perpendicular distance from (0, 0) to $x + y = 1$ is $\frac{1}{\sqrt{2}}$.

$\therefore$ $AO = \sqrt{2} \Rightarrow A(-1, -1)$

If m is the slope of other side, then

$$\tan 60° = \left|\frac{m+1}{1-m}\right| \Rightarrow m = 2 \pm \sqrt{3}$$

19. (a) : $T = S_1$

i.e., $xx_1 + yy_1 - (y + y_1) = x_1^2 + y_1^2 - 2y_1$

Passes through (0, 0).

$\therefore$ $x^2 + y^2 - y = 0$

Drop in JEE Main 2020 registrations; EWS and women applicants on the rise

National Testing Agency (NTA) has ended the registration process for JEE Main 2020 on October 10, 2019. With each year witnessing a huge number of registrations for the national level engineering entrance examination, JEE Main 2020 has experienced a small drop in the numbers. While the previous year had 9.41 lakh registered candidates, NTA has reportedly received around 9.34 lakh registrations for JEE Main 2020.

According to reports, Economically Weaker Section (EWS) category, which was introduced this year, has over 81,413 registrations. The huge number of registrations for the newly created category has caused a decline in the open category registrations by almost 90,000 candidates.

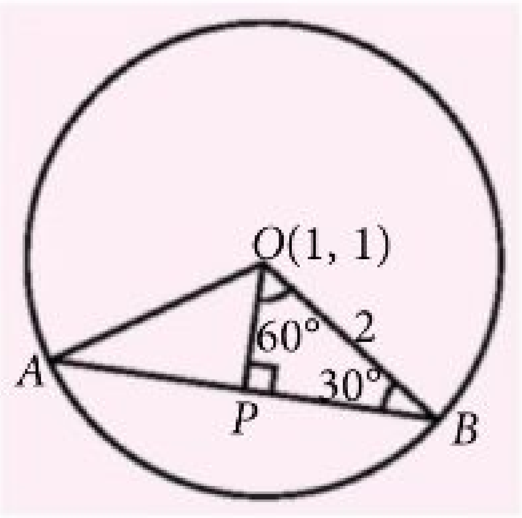
Along with EWS, the number of women applicants for JEE Main 2020 has also increased. As compared to the previous year, 15,247 more women have applied for the entrance examination. In total, around 2.90 lakh women candidates are expected to attempt JEE Main 2020 January session. For B.Arch/Planning, NTA has reportedly received 1.38 lakh registrations for the January session. While the drop in the registration numbers is not severe, there has been a constant decline in the number of applicants since the introduction of two JEE Main attempts. While the authorities are trying their best to attract women students into the institutes, the total number of registrations is still on decline.

Year	Total Number of Registrations
2020	9.34 lakh
2019	9.41 lakh
2018	11.40 lakh
2017	11.86 lakh
2016	12.07 lakh
2015	13.04 lakh
2014	13.56 lakh

20. (a) : Centre (1, 1) and radius = 2 = OB

Now, $\angle OBP = 30°$

$$\therefore\ \sin 30° = \frac{OP}{2} \Rightarrow OP = 1$$



Since, $OP = 1$

$\therefore\ x^2 + y^2 - 2x - 2y + 1 = 0$

21. (3) : $f(1, 1) = f(0, f(1, 0)) = f(0, f(0, 1)) = f(0, 2) = 3$

22. (4) : $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$ ...(i)

Now, $\sin x > 0$ and $\cos x > 0$

$\therefore$ From (i), $\sin x \cos x = \frac{1}{2} \Rightarrow \sin 2x = 1$

$\Rightarrow$ 4 solutions $[\because\ 2x \in [-4\pi, 4\pi]]$

23. (0) : Let $y = f(x) = \dfrac{x-1}{c - x^2 + 1}$

Take $y = -t$, where $t \in \left[\frac{1}{3}, 1\right]$ $\therefore\ -t = \dfrac{x-1}{c - x^2 + 1}$

$$\Rightarrow\ x^2 - c - 1 = \frac{x-1}{t} \Rightarrow x^2 - \frac{1}{t}x + \frac{1}{t} - c - 1 = 0$$

As $-t \in \left[-1, -\frac{1}{3}\right]$, hence the above must not possess real solution.

$$\therefore\ \left(\frac{1}{t}\right)^2 - 4\left(\frac{1}{t} - c - 1\right) < 0 \Rightarrow \frac{1}{t^2} - \frac{4}{t} + 4 < -4c$$

$$\Rightarrow\ c < -\frac{1}{4}\left(\frac{1}{t} - 2\right)^2$$

Now, $\frac{1}{3} \le t \le 1 \Rightarrow -1 \le \frac{1}{t} - 2 \le 1 \Rightarrow 0 \le \frac{1}{4}\left(\frac{1}{t} - 2\right)^2 \le \frac{1}{4}$

Hence, $c \in \left(-\infty, -\frac{1}{4}\right)$

24. (256)

25. (0) : Given that $\sin^5 x - \cos^5 x = \dfrac{\sin x - \cos x}{\sin x \cos x}$

$$\Rightarrow\ \sin x \cos x \left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$$\Rightarrow\ \sin x \cos x\{(\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x + \sin x \cos x\} = 1$$

$$\Rightarrow\ \frac{1}{2}\sin 2x\left[1 - \frac{1}{4}\sin^2 2x + \frac{1}{2}\sin 2x\right] = 1$$

$\Rightarrow\ \sin 2x\ (\sin^2 2x - 2\sin 2x - 4) = -8$

$\Rightarrow\ \sin^3 2x - 2\sin^2 x - 4\sin 2x + 8 = 0$

$\Rightarrow\ (\sin 2x - 2)^2\ (\sin 2x + 2) = 0$

$\Rightarrow\ \sin 2x = \pm 2$, which is impossible.

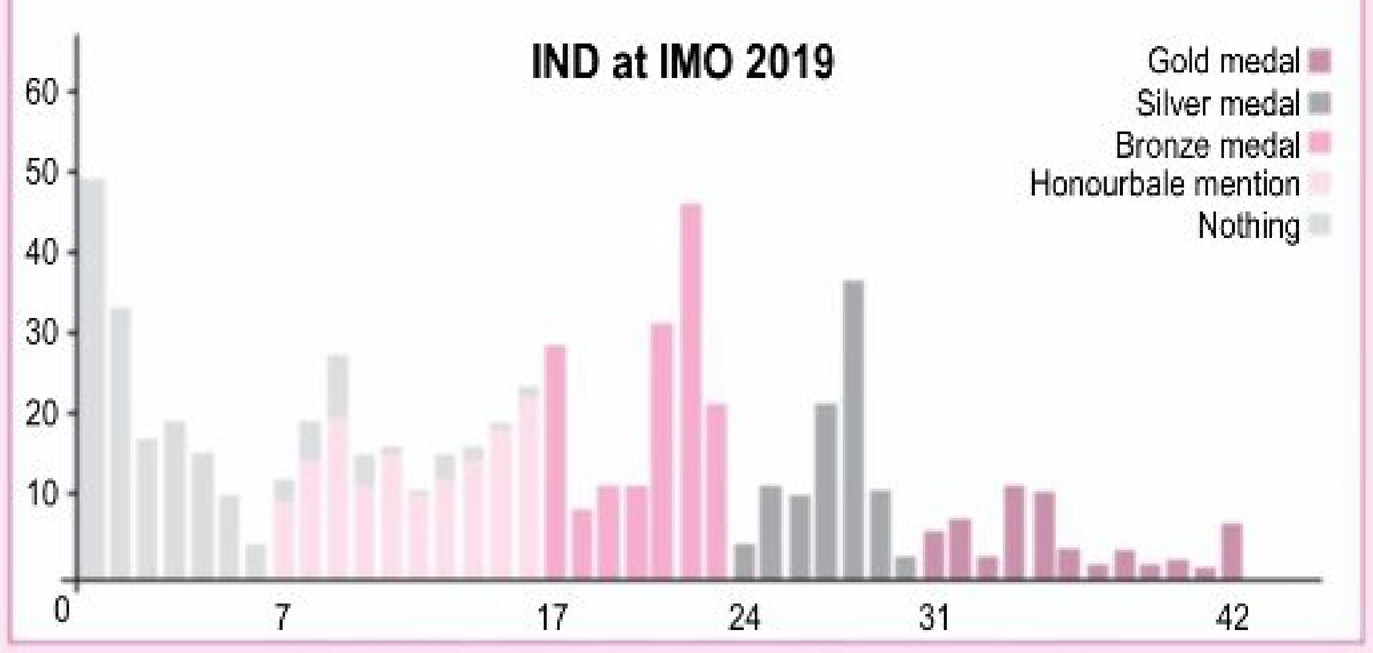
◆◆

15-year-old Bengaluru boy Pranjal Srivastava has become the youngest Indian to bag gold medal in IMO

Bengaluru boy Pranjal Srivastava has become the youngest Indian ever to bag a gold medal at the International Mathematics Olympiad, one of the world's toughest competitions.

IMO 2019 result details for the India participants given here:

◀ **60TH IMO 2019**



Contestant (25)	P1	P1	P3	P4	P5	P6	Total	Rank		Award
								Abs.	Rel.	
Pranjal Srivastava	7	6	7	7	7	1	35	18	97.26%	Gold medal
Ritam Nag	7	6	0	7	7	0	27	101	83.87%	Silver medal
Anubhav Ghosal	7	6	0	7	7	0	27	101	83.87%	Silver medal
Bhavya Agrawalla	7	6	0	7	7	0	27	101	83.87%	Silver medal
Ojas Mittal	7	7	0	7	3	0	24	143	77.10%	Silver medal
Soumil Aggarwal	7	1	0	6	2	0	16	303	51.29%	Honourable mention
Team results	**42**	**32**	**7**	**41**	**33**	**1**	**156**	**15**	**87.39%**	**G, S, S, S, S, HM**

This was after seven years that India secured a gold medal in the prestigious Maths Olympiad held in the United Kingdom this year. Pranjal is a student of National Public School, Koramangala. The International Mathematics Olympiad (IMO) which was termed "as the biggest and toughest of competitions across the world" by CBSE had participants from 210 countries and more than 600 participants. Pranjal participated in IMO 2019 with 5 other team members and secured a relative ranking quotient of 97.26%.

Considering the subject a 'fun activity', Pranjal has been participating in math competitions since class 3. This was his second attempt at IMO, last year, he won the silver medal. Pranjal thinks it is his determination to understand the subject over "mugging" that gives him an edge. A change in approach can help India get over its math phobia, believes the child mathematician. The child mathematician was also the first Indian to grab a gold in the Asia Pacific Mathematics Olympiad and has won the Tournament of Towns along with other competitions. He was also among the national toppers of Olympiads in informatics but chose to pursue the math Olympiad instead.

◆◆

MOCK TEST PAPER 2020
JEE MAIN

with
Numerical Value
Type Questions

Exam Date : Between 6th to 11th January

Time : 1 hr 15 min.

1. A letter is taken out at random from 'ASSISTANT' and another letter taken out from the letters of the word 'STATISTICS'. The probability that they are identical letters, is

(a) $\frac{13}{90}$ (b) $\frac{1}{45}$

(c) $\frac{19}{90}$ (d) none of these

2. If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and $B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}$, then $A \cap B =$

(a) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\}$ (b) $\left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

(c) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

(d) none of these

3. If $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$ $= A\tan^{-1}(B\tan x + C) + C_1$ then

(a) $A = \frac{1}{4}, B = \frac{1}{2}, C = 1$ (b) $A = \frac{1}{2}, B = \frac{1}{4}, C = 1$

(c) $A = 1, B = \frac{1}{2}, C = \frac{1}{4}$ (d) $A = \frac{1}{4}, B = 1, C = \frac{1}{2}$

4. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $(A(\text{adj } A)A^{-1})A =$

(a) $2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1/6 & -1/6 \\ 2/6 & 1/6 & 3/6 \\ 3/6 & 2/6 & 1/6 \end{bmatrix}$ (d) none of these

5. If X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y|X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$ then which of the following is incorrect?

(a) $P(X \cup Y) = \frac{2}{3}$ (b) X and Y are independent

(c) $P(X^c \cap Y) = \frac{1}{6}$

(d) X and Y are not independent

6. Let $\vec{a} = \alpha_1\hat{i} + \alpha_2\hat{j} + \alpha_3\hat{k}$, $\vec{b} = \beta_1\hat{i} + \beta_2\hat{j} + \beta_3\hat{k}$ and $\vec{c} = \gamma_1\hat{i} + \gamma_2\hat{j} + \gamma_3\hat{k}$, $|\vec{a}| = 2\sqrt{2}$, $\vec{a}$ makes angle $\frac{\pi}{3}$ with plane of $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}^n$ is equal to (n is even natural number)

(a) $\left(\frac{|\vec{b}||\vec{a}|}{6}\right)^{n/2}$ (b) $\left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$

(c) $\frac{\left(|\vec{b}||\vec{c}|\right)^{n/2}}{\sqrt{32^n}}$ (d) none of these

7. If α, β are roots of the equation $p(x^2 - x) + x + 5 = 0$ and p_1, p_2 are two values of p for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$ then the value of $\frac{p_1}{p_2} + \frac{p_2}{p_1} =$

(a) 254 (b) 0 (c) 245 (d) –254

8. If $f(x) = \begin{cases} \frac{a|x^2 - x - 2|}{2 + x - x^2}, & x < 2 \\ b, & x = 2 \\ \frac{x - [x]}{x - 2}, & x > 2 \end{cases}$ is continuous at $x = 2$, then

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

(a) $a = 1, b = 2$ (b) $a = b = 1$

(c) $a = 2, b = 1$ (d) $a = b = 2$

9. If $f(x) = |x|^{|\tan x|}$, then $f'\left(-\dfrac{\pi}{6}\right) =$

(a) $\left(\dfrac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\dfrac{2\sqrt{3}}{\pi} - \dfrac{4}{3}\log\dfrac{6}{\pi}\right\}$

(b) $\left(\dfrac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\dfrac{2\sqrt{3}}{\pi} + \dfrac{4}{3}\log\dfrac{6}{\pi}\right\}$

(c) $\left(\dfrac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\dfrac{2\sqrt{3}}{\pi} + \dfrac{4}{3}\log\dfrac{6}{\pi}\right\}$

(d) none of these

10. If the sum of the coefficients in the expansion of $(b + c)^{20}\,[1 + (a - 2)x]^{20}$ is equal to square of the sum of the coefficients in the expansion of $[2bc\,x - (b + c)y]^{10}$, where a, b, c are positive constants, then

(a) $a \ge \sqrt{bc}$ (b) $\dfrac{b+c}{2} \ge a$

(c) $a^2 = cb$ (d) $\dfrac{1}{c}, \dfrac{1}{a}, \dfrac{1}{b}$ are in G.P.

11. The tangent at a point $P(a\cos\theta, b\sin\theta)$ on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the auxiliary circle in two points. The chord joining them subtends a right angle at the centre. Then, the eccentricity of the ellipse is given by

(a) $(1+\sin^2\theta)^{-1/2}$ (b) $(1+\cos^2\theta)^{-1/2}$

(c) $(1 + \sin^2\theta)$ (d) $(1+\cos^2\theta)^{1/2}$

12. If $[x]$ stands for the greatest integer function, then value of $\displaystyle\int_4^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]}$ is

(a) 0 (b) 1

(c) 3 (d) none of these

13. If $(x^2 + y^2)dy = xydx$, $y(1) = 1$, and $y(x_0) = e$ then $x_0 =$

(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$

(c) $\sqrt{3}e$ (d) $\sqrt{\dfrac{e^2 + 1}{2}}$

14. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is

(a) 0.533 (b) 0.466

(c) 0.763 (d) none of these

15. Square root(s) of $-i$ is/are

(a) $\dfrac{1}{\sqrt{2}}(1 - i)$ (b) $\dfrac{1}{\sqrt{3}}(i - 1)$

(c) $\pm\dfrac{1}{2}(1 - i)$ (d) $-\dfrac{1}{\sqrt{2}}(1 - i)$

16. If $\tan\alpha = \dfrac{x^2 - x}{x^2 - x + 1}$ and $\tan\beta = \dfrac{1}{2x^2 - 2x + 1}$, $0 < \alpha, \beta < \dfrac{\pi}{2}$, then $\alpha + \beta =$

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{3\pi}{4}$

17. Let R be a relation defined on the set Z of all integers such that $xRy \Leftrightarrow x + 2y$ is divisible by 3. Then

(a) R is transitive only

(b) R is symmetric only

(c) R is an equivalence relation

(d) R is not an equivalence relation

18. Let $A = \{x \in R : -1 \le x \le 1\} = B$. Then the mapping $f : A \to B$ given by $f(x) = x|x|$ is

(a) injective but not surjective

(b) surjective but not injective

(c) bijective (d) none of these

19. The lines $\dfrac{x-2}{1} = \dfrac{y-3}{1} = \dfrac{z-4}{-k}$ and $\dfrac{x-1}{k} = \dfrac{y-4}{2} = \dfrac{z-5}{1}$ are coplanar, if

(a) $k = 3$ or -3 (b) $k = 0$ or -1

(c) $k = 1$ or -1 (d) $k = 0$ or -3

20. If a variable straight line passes through the intersection of the lines $x + 2y = 1$ and $2x - y = 1$ and meet the co-ordinate axes at A and B, then the locus of mid point of AB is

(a) $x - 3y = 10xy$ (b) $x + 3y = 10xy$

(c) $3x + y = 10xy$ (d) none of these

NUMERICAL VALUE TYPE

21. If $y = \sin(2\sin^{-1}x)$, then the value of $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 4y$ is

22. The area (in sq. units) of the quadrilateral formed by the tangents at the end-points of latus rectum of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ is

23. A spherical balloon is being inflated at the rate of 35cm^3/min. Then, the rate of increase of surface area (in cm^2 /min) of the balloon when its diameter is 14 cm is equal to

24. The first of the two samples has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second group is equal to

25. The number of integral values of k for which the equation $3\cos x + 4\sin x = 2k + 1$ has a solution, is

SOLUTIONS

1. (c) : In the word 'ASSISTANT' there are SSS AA TT I N and in 'STATISTICS" there are A C II SSS TTT. The non common letters are N, C. The identical letters are A, I, S, T.

The probability of getting $A = \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^1C_1}{{}^{10}C_1} = \frac{1}{45}$,

probability of getting $I = \frac{{}^1C_1}{{}^9C_1} \times \frac{{}^2C_1}{{}^{10}C_1} = \frac{1}{45}$,

probability of getting $S = \frac{{}^3C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{1}{10}$, and

probability of getting $T = \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{1}{15}$

$\therefore$ Required probability $= \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$

2. (c) : We have, $2\cos^2\theta + \sin\theta \le 2$

$\Rightarrow 2(1 - \sin^2\theta) + \sin\theta \le 2 \Rightarrow 2\sin^2\theta - \sin\theta \ge 0$

$\Rightarrow \sin\theta\,(2\sin\theta - 1) \ge 0 \Rightarrow \sin\theta \ge 0$ and $(2\sin\theta - 1) \ge 0$

or $\sin\theta \le 0$ and $(2\sin\theta - 1) \le 0$

Case I : When $\sin\theta \ge 0$ and $(2\sin\theta - 1) \ge 0$

$$\Rightarrow \sin\theta \ge 0 \text{ and } \sin\theta \ge \frac{1}{2} \Rightarrow \frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$$

$$\therefore A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

Case II : When $\sin\theta \le 0$ and $(2\sin\theta - 1) \le 0$

$$\Rightarrow \sin\theta \le 0 \text{ and } \sin\theta \le \frac{1}{2} \Rightarrow \pi \le \theta \le 2\pi$$

$$\therefore A \cap B = \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\} \left[\because B = \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right]$$

Thus, $A \cap B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\} \cup \left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$.

3. (d) : Here, $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$

$$= \int \frac{\sec^2 x\,dx}{4\tan^2 x + 4\tan x + 5} = \int \frac{dz}{(2z+1)^2 + 2^2},$$

where $z = \tan x$

$$= \frac{1}{4} \cdot \tan^{-1}\left(\frac{2z+1}{2}\right) + C_1 = \frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + C_1$$

$$= A\tan^{-1}(B\tan x + C) + C_1$$

Therefore, $A = \frac{1}{4}$, $B = 1$ and $C = \frac{1}{2}$

4. (a) : We have, $|A| = \begin{vmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 + 7 - 1 = 6$

$$\therefore (A(\text{adj}A)A^{-1})A = (A(\text{adj}A))(A^{-1}A)$$
$$= (|A|I)I$$
$$= |A|I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

5. (d) : We have, $P(X\,|\,Y) = \frac{1}{2}$, $P(Y\,|\,X) = \frac{1}{3}$

and $P(X \cap Y) = \frac{1}{6}$

$$\therefore P(X) = \frac{P(X \cap Y)}{P(Y\,|\,X)} = \frac{1/6}{1/3} = \frac{1}{2},$$

$$P(Y) = \frac{P(X \cap Y)}{P(X\,|\,Y)} = \frac{1/6}{1/2} = \frac{1}{3}$$

Clearly, X and Y are independent events.

$$\therefore P(X \cup Y) = 1 - P(\bar{X})P(\bar{Y}) = 1 - \frac{1}{2} \times \frac{2}{3} = \frac{2}{3}.$$

6. (b) : Given determinant shows the volume of parallelepiped made by vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

$$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = |\vec{a}||\vec{b} \times \vec{c}|\sin\frac{\pi}{3} = |\vec{a}||\vec{b}||\vec{c}|\sin\frac{\pi}{3}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}} \quad [\because \ |\vec{a}| = 2\sqrt{2}]$$

7. (a) : Here, $p(x^2 - x) + x + 5 = 0$

$\Rightarrow px^2 - (p - 1)x + 5 = 0$

$$\therefore \alpha + \beta = \frac{p-1}{p} \text{ and } \alpha\beta = \frac{5}{p}$$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{5}$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5} \Rightarrow \frac{(p-1)^2 - 10p}{5p} = \frac{4}{5}$$

$\Rightarrow p^2 - 16p + 1 = 0 \ \therefore\ p_1 + p_2 = 16$ and $p_1p_2 = 1$

Now, $\frac{p_1}{p_2} + \frac{p_2}{p_1} = \frac{(p_1 + p_2)^2 - 2p_1p_2}{p_1p_2} = \frac{256 - 2}{1} = 254$

8. (b) : Since in the neighbourhood of 2 on left side $|x^2 - x - 2| = -(x^2 - x - 2)$ and $x - [x] = x - 2$ if $x \in (2, 3)$

Thus, the function can be redefined as $f(x) = \begin{cases} a \text{ if } x < 2 \\ b \text{ if } x = 2 \\ 1 \text{ if } x > 2 \end{cases}$

$\therefore$ $f(x)$ is continuous at $x = 2 \Rightarrow a = b = 1$

9. (b) : We have, $f(x) = |x|^{|\tan x|}$

$\Rightarrow$ $f(x) = (-x)^{-\tan x}$ [$\because$ In the neighbourhood of $x = -\dfrac{\pi}{6}, |x| = -x$ and $|\tan x| = -\tan x$]

$\Rightarrow$ $\log f(x) = -\tan x \log(-x)$

Differentiating w.r.t. x, we get

$$\frac{1}{f(x)} \cdot f'(x) = \left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow \quad f'(x) = (-x)^{-\tan x}\left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow \quad f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\frac{4}{3}\log\frac{\pi}{6} - \frac{2\sqrt{3}}{\pi}\right\}$$

10. (b) : The sum of the coefficients in the expansion of $(b + c)^{20}\,[1 + (a - 2)x]^{20}$ is given by

$S_1 = (b + c)^{20}\,(a - 1)^{20}$ [Putting $x = 1$]

The sum of the coefficients in the expansion of $[2bcx - (b + c)y]^{10}$ is given by

$S_2 = [2bc - (b + c)]^{10}$ [Putting $x = y = 1$]

It is given that $S_1 = S_2^2$

$\Rightarrow$ $(b + c)^{20}\,(a - 1)^{20} = [2bc - (b + c)]^{20}$

$\Rightarrow$ $(b + c)(a - 1) = [2bc - (b + c)]$

$\Rightarrow$ $(a-1) = \dfrac{2bc}{b+c} - 1 \Rightarrow a = \dfrac{2bc}{b+c}$

$\Rightarrow$ a is the H.M. of b and c.

Therefore, A.M. ≥ H. M. $\Rightarrow \dfrac{b+c}{2} \geq a$.

11. (a)

12. (c) : Let $I = \displaystyle\int_4^{10} \frac{[x^2]dx}{[x^2 - 28x + 196] + [x^2]}$

$$= \int_4^{10} \frac{[x^2]dx}{[(14-x)^2] + [x^2]} = \int_4^{10} \frac{(14-x)^2 dx}{[x^2] + [(14-x)^2]}$$

$$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$\therefore$ $2I = \displaystyle\int_4^{10} dx = 6 \Rightarrow I = 3$

13. (c) : We have, $\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$

Putting $y = vx$, we get $v + x\dfrac{dv}{dx} = \dfrac{v}{1+v^2}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v}{1+v^2} - v = -\frac{v^3}{1+v^2}$$

$$\Rightarrow \quad \frac{1+v^2}{v^3}dv = -\frac{dx}{x} \Rightarrow \int \frac{1+v^2}{v^3}dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \ln x + \ln v - \frac{1}{2v^2} = c \Rightarrow \ln y = \frac{x^2}{2y^2} + c$$

Now, $x = 1, y = 1 \Rightarrow c = -\dfrac{1}{2}$

Again, $y = e \Rightarrow 1 = \dfrac{x_0^2}{2e^2} - \dfrac{1}{2} \Rightarrow x_0^2 = 3e^2 \Rightarrow x_0 = \sqrt{3}e$

14. (a)

15. (a) : We have, $-i = \dfrac{1}{2}(-2i) = \dfrac{1}{2}(1^2 + i^2 - 2i)$

$$= \frac{1}{2}(1-i)^2 \quad \therefore \quad \sqrt{-i} = \frac{1}{\sqrt{2}}(1-i)$$

16. (a) : $\tan\alpha = \dfrac{x^2 - x}{x^2 - x + 1}$ and $\tan\beta = \dfrac{1}{2x^2 - 2x + 1}$, $0 < \alpha, \beta < \dfrac{\pi}{2}$

Let $x^2 - x = m$ $\therefore$ $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$

$$\text{Now, } \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{(m+1)(2m+1)}}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{1}{m+1} - \frac{1}{2m+1}\right)} = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{\frac{m}{m+1} + \frac{1}{2m+1}} = 1$$

Hence, $\alpha + \beta = \dfrac{\pi}{4}$ (As $0 < \alpha, \beta < \dfrac{\pi}{2}$)

17. (c) : Let $(x, x) \in Z$, then $x + 2x = 3x$ is divisible by 3. $\therefore$ R is Reflexive.

If $x + 2y$ is divisible by 3 then $y + 2x$ is also divisible by 3. $\therefore$ R is symmetric.

Let xRy and yRz for some $x, y, z \in Z$

Let $x + 2y = 3k_1$ and $y + 2z = 3k_2 \Rightarrow x + 3y + 2z = 3(k_1 + k_2) \Rightarrow x + 2z = 3(k_1 + k_2 - y)$, which is divisible by 3.

$\therefore$ R is transitive. So, R is an equivalence relation.

18. (c) : Let, $x_1, x_2 \in R$ $\therefore f(x_1) = f(x_2) \Rightarrow x_1|x_1| = x_2|x_2|$

$\therefore$ Mapping f is one-to-one. So, $x_1 = x_2$

Again, let $y \in R$ such that $y = f(x) \Rightarrow y = x|x|$

Now, when $-1 \leq x \leq 1$ then $-1 \leq y \leq 1$

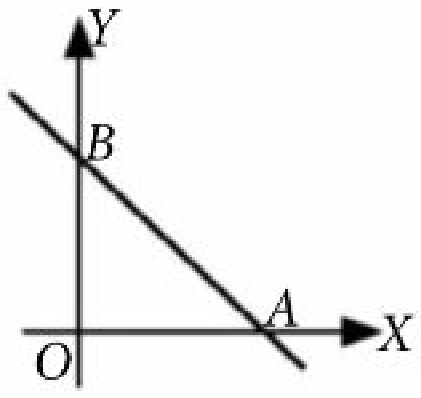
$\therefore$ Range and co-domain of f are equal.

Thus, f is onto. Therefore f is one-to-one and onto, hence f is bijective.

19. (d) : So, the given lines will be coplanar iff

$$\begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow k^2 + 3k = 0 \Rightarrow k = 0, -3$$

20. (b) : The equation of the straight line passing through the point of intersection of the lines $x + 2y = 1$ and $2x - y = 1$ is



$$(x + 2y - 1) + \lambda(2x - y - 1) = 0$$

$$\Rightarrow (1 + 2\lambda)x + (2 - \lambda)y = (1 + \lambda)$$

$$\Rightarrow \frac{x}{\frac{1+\lambda}{1+2\lambda}} + \frac{y}{\frac{1+\lambda}{2-\lambda}} = 1$$

$$\therefore \quad A \equiv \left(\frac{1+\lambda}{1+2\lambda}, 0\right) \text{ and } B \equiv \left(0, \frac{1+\lambda}{2-\lambda}\right)$$

Let the mid-point of AB be (h, k).

$$\therefore \quad h = \frac{1}{2}\left(\frac{1+\lambda}{1+2\lambda}\right) \text{ and } k = \frac{1}{2}\left(\frac{1+\lambda}{2-\lambda}\right)$$

Eliminating λ from h and k we get $h + 3k = 10hk$

Therefore, the required locus is $x + 3y = 10xy$.

21. (0) : We have, $y = \sin(2\sin^{-1}x)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}\cos(2\sin^{-1}x)$$

$$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = 2\cos(2\sin^{-1}x)$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4\{1-\sin^2(2\sin^{-1}x)\} = 4 - 4y^2$$

Again differentiating w.r.t. x, we get

$$2(1-x^2)\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -8y\frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$$

22. (27) : The equation of the ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$$\therefore \quad \text{The eccentricity, } e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$$

The coordinates of the end-points of latusrecta are

$$\left(2, \frac{5}{3}\right), \left(2, -\frac{5}{3}\right), \left(-2, \frac{5}{3}\right), \left(-2, -\frac{5}{3}\right).$$

The equations of tangents at these points are

$2x + 3y - 9 = 0$ ...(i)

$-2x + 3y + 9 = 0$ ...(ii)

$-2x + 3y - 9 = 0$ ...(iii)

$2x + 3y + 9 = 0$ ...(iv)

Clearly the above tangents form a parallelogram whose area is given by

$$A = \left|\frac{\{9-(-9)\}\times\{9-(-9)\}}{\begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix}}\right| = \frac{18\times 18}{12} = 27 \text{ sq.units}$$

23. (10) : Let the volume, surface area and radius of the spherical balloon be v, s and r respectively.

$$\therefore \quad v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\text{Again, } s = 4\pi r^2 \Rightarrow \frac{ds}{dt} = 8\pi r\frac{dr}{dt}$$

$$\text{Now, } \frac{\frac{ds}{dt}}{\frac{dv}{dt}} = \frac{2}{r} \Rightarrow \frac{ds}{dt} = \frac{2}{r}\cdot\frac{dv}{dt} = \frac{2}{7}\times 35 = 10$$

$$\left[\because r = 7 \text{ cm and } \frac{dv}{dt} = 35 \text{ cm}^3/\text{min}\right]$$

Thus, the rate of increase of the surface area is 10 cm^2/min, when diameter of the balloon is 14 cm.

24. (4) : We have, $n_1 = 100$, $\bar{x}_1 = 15$, $\sigma_1 = 3$, $n = n_1 + n_2 = 250$, $\bar{x} = 15.6$ and $\sigma = \sqrt{13.44}$

Now, $n_2 = 250 - 100 = 150$

$$\text{We know, } \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow 15.6 = \frac{100\times 15 + 150\times\bar{x}_2}{250} \Rightarrow \bar{x}_2 = 16$$

Hence, $d_1 = \bar{x}_1 - \bar{x} = 15 - 15.6 = -0.6$ and $d_2 = \bar{x}_2 - \bar{x}$ $= 16 - 15.6 = 0.4$

The variance σ^2 of the combined group is given by the formula :

$$(n_1 + n_2)\sigma^2 = n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)$$

$$\Rightarrow 250 \times 13.44 = 100(9 + 0.36) + 150(\sigma_2^2 + 0.16)$$

$$\Rightarrow 150\sigma_2^2 = 250 \times 13.44 - 100 \times 9.36 - 150 \times 0.16 = 2400$$

$$\Rightarrow \sigma_2^2 = \frac{2400}{150} = 16 \Rightarrow \sigma_2 = \sqrt{16} = 4$$

25. (6) : The given equation is $3\cos x + 4\sin x = 2k + 1$

$$\Rightarrow \frac{3}{\sqrt{3^2+4^2}}\cos x + \frac{4}{\sqrt{3^2+4^2}}\sin x = \frac{2k+1}{\sqrt{3^2+4^2}}$$

$$\Rightarrow \frac{3}{5}\cos x + \frac{4}{5}\sin x = \frac{2k+1}{5}$$

$$\Rightarrow \cos(x-\alpha) = \frac{2k+1}{5} \left(\text{where } \cos\alpha = \frac{3}{5}\right)$$

$$\Rightarrow -1 \le \frac{2k+1}{5} \le 1 \qquad [\because -1 \le \cos(x-\alpha) \le 1]$$

$$\Rightarrow -5 \le 2k + 1 \le 5 \Rightarrow -6 \le 2k \le 4 \Rightarrow -3 \le k \le 2$$

Therefore integral values of k are $-3, -2, -1, 0, 1, 2$.

❖❖

Challenging PROBLEMS

ON

Vectors and 3D Geometry

1. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r}=\vec{b}+\lambda\vec{c}$ is

(a) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$ (b) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}|}$

(c) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$ (d) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}$

2. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{c}+\vec{b}$, then

(a) $[\vec{a}\,\vec{b}\,\vec{c}]=\vec{b}\cdot\vec{c}-(\vec{a}\cdot\vec{b})^2$

(b) $[\vec{a}\,\vec{b}\,\vec{c}]=0$

(c) Maximum value of $[\vec{a}\,\vec{b}\,\vec{c}]$ is 1

(d) Minimum value of $[\vec{a}\,\vec{b}\,\vec{c}]$ is $\dfrac{1}{2}$

3. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$, then which of the following is always true?

(a) $\vec{a},\vec{b},\vec{c},\vec{d}$ are necessarily coplanar

(b) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ or $\vec{c}$

(c) either $\vec{b}$ or $\vec{c}$ must lie in plane of $\vec{a}$ and $\vec{d}$

(d) either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

4. The lengths of two opposite edges of a tetrahedron are a, b. Their shortest distance is d and the angle between them is θ. Then its volume, V is

(a) $\dfrac{1}{2}abd\sin\theta$ (b) $\dfrac{1}{3}abd\cos\theta$

(c) $\dfrac{1}{6}abd\sin\theta$ (d) $\dfrac{1}{6}abd\cos\theta$

5. The position vectors of the vertices A, B, C of a tetrahedron $ABCD$ are $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}$ and $3\hat{i}$ respectively and the altitude from the vertex D to the opposite face ABC meets the face at E. If the length of the edge AD is 4 and the volume of the tetrahedron is $\dfrac{2\sqrt{2}}{3}$, then the length of DE is

(a) 1 (b) 2 (c) 3 (d) 4

6. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}\cdot\vec{b}=0$, $(\vec{a}-\vec{c})\cdot(\vec{b}+\vec{c})=0$ and $\vec{c}=\lambda\vec{a}+\mu\vec{b}+\omega(\vec{a}\times\vec{b})$, where λ,μ,ω are scalars, then

(a) $\mu^2+\omega^2=1$

(b) $\lambda+\mu=1$

(c) $(\mu+1)^2+\mu^2+\omega^2=1$

(d) $\lambda^2+\mu^2=1$

7. If $\vec{u},\vec{m},\vec{r}$ be three mutually perpendicular vectors with same magnitude. If $\vec{e}$ satisfies the relation $\vec{u}\times\{(\vec{e}-\vec{m})\times\vec{u}\}+\vec{m}\times\{(\vec{e}-\vec{r})\times\vec{m}\}+\vec{r}\times\{(\vec{e}-\vec{u})\times\vec{r}\}=\vec{0}$ then $\vec{e}=$

(a) $\dfrac{1}{3}(\vec{u}+\vec{m}+\vec{r})$ (b) $\dfrac{1}{2}(\vec{u}+\vec{m}+\vec{r})$

(c) $\dfrac{1}{4}(\vec{u}+\vec{m}+\vec{r})$ (d) $\vec{0}$

8. If $\vec{a},\vec{b}$ and $\vec{c}$ are three non coplanar unit vectors each inclined with other at an angle of 30°, then the volume of tetrahedron whose edges are $\vec{a},\vec{b},\vec{c}$ is

(a) $\dfrac{\sqrt{3\sqrt{3}-5}}{12}$ (b) $\dfrac{3\sqrt{3}+5}{12}$

(c) $\dfrac{5\sqrt{2}+3}{12}$ (d) $\dfrac{3\sqrt{3}}{8}$

9. If θ is the angle between any edge and a face not containing the edge of a regular tetrahedron, then the value of $\cos\theta$ is

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{3}$ (c) $\sqrt{\dfrac{2}{3}}$ (d) $\dfrac{\sqrt{2}}{3}$

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10. In a triangle ABC, $\angle A = 30°$, H is the orthocenter and D is the midpoint of BC. If the segment HD is produced to T such that $HD = DT$. Then $\frac{AT}{BC} =$

(a) 1/2 (b) 3/2 (c) 2 (d) 2/3

11. Let position vectors of points A, B and C of triangle ABC respectively be $\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$. Let l_1, l_2, l_3 be the lengths of perpendiculars drawn from the orthocentre 'O' on the sides AB, BC and CA, then $l_1 + l_2 + l_3$ equals

(a) $\frac{\sqrt{6}}{3}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{\sqrt{6}}{2}$

12. Three vectors of magnitude a, $2a$, $3a$ meet in a point and their directions are along the diagonals of three adjacent faces of a cube. Then the magnitude of their resultant is

(a) $6a$ (b) $4a$ (c) $3a$ (d) $5a$

13. Let $\vec{a}$ and $\vec{b}$ be two non collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{v} = \vec{a}\times\vec{b}$ then $|\vec{v}| =$

(a) $|\vec{u}|$ (b) $|\vec{u}| + |\vec{u}\cdot\vec{a}|$

(c) $2|\vec{u}| + |\vec{u}\cdot\vec{b}|$ (d) $|\vec{u}| + \vec{u}\cdot(\vec{a}+\vec{b})$

14. In a four-dimensional space where unit vectors along axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{l}$ and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non zero vectors such that no vector can be expressed as linear combination of others and $(\lambda-1)(\vec{a}_1-\vec{a}_2)$ $+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{0}$, then

(a) $\lambda = 1$ (b) $2\mu = -2/3$

(c) $\lambda = 2/3$ (d) $2\delta = 1/3$

15. Let $OABC$ be a tetrahedron whose four faces are equilateral triangles of unit side. Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$, then

(a) $\vec{c} = \frac{1}{3}(\vec{a}+\vec{b}+2\sqrt{2}(\vec{a}\times\vec{b}))$

(b) $\vec{c} = \frac{1}{2}(\vec{a}+\vec{b}+2\sqrt{3}\,(\vec{a}\times\vec{b}))$

(c) volume of the tetrahedron is $\frac{1}{2\sqrt{3}}$

(d) $2^{\|[\vec{a}\;\vec{b}\;\vec{c}]\|} = \frac{1}{\sqrt{2}}$

16. $OABC$ is regular tetrahedron of unit edge length with volume V then $12\sqrt{2}V =$

(a) 1 (b) 2 (c) 1/2 (d) 1/3

17. ABC is any triangle and O is any point in the plane of ΔABC. If AO, BO and CO meet the sides BC, CA and AB in D, E, F respectively, then $\frac{OD}{AD}+\frac{OE}{BE}+\frac{OF}{CF}=$

(a) 1 (b) 2 (c) 1/2 (d) 1/3

18. Let ABC be a given triangle. If $|\overrightarrow{BA} - t\overrightarrow{BC}| \geq |\overrightarrow{AC}|$ for all $t \in R$. Then ΔABC is

(a) equilateral (b) right angled

(c) isosceles (d) none of these

19. Let $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then the value of

$$\begin{vmatrix} {c_1}^2 & a_1b_2-a_2b_1+c_1c_2 & a_1b_3-a_3b_1+c_1c_3 \\ a_2b_1-a_1b_2+c_1c_2 & {c_2}^2 & a_2b_3-a_3b_2+c_3c_2 \\ a_3b_1-a_1b_3+c_1c_3 & a_3b_2-a_2b_3+c_3c_2 & {c_3}^2 \end{vmatrix} \text{ is}$$

(a) 0 (b) 1

(c) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

(d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

20. In ΔABC, perpendiculars are drawn from A, B, C to meet BC, CA, AB at D, E, F respectively. If $\angle A = 75°$, $\angle B = 60°$ and $BC = 4(\sqrt{3}+1)$, then value of $\left|\frac{DF(\overrightarrow{DE})+DE(\overrightarrow{DF})}{DE+DF+EF}\right|$ equals

(a) 2 (b) 4 (c) 6 (d) 8

21. Let O be the origin, P be a given point and M, N are projections of P on ZX and XY planes respectively. If $\overrightarrow{OP}$ makes angles $\theta, \alpha, \beta, \gamma$ with the plane OMN and the coordinate planes YZ, ZX, XY respectively, then $\cot^2\theta =$

(a) $\cot^2\alpha - \cot^2\beta - \cot^2\gamma - 2$

(b) $-\cot^2\alpha + \cot^2\beta + \cot^2\gamma - 1$

(c) $\cot^2\alpha + \cot^2\beta + \cot^2\gamma + 2$

(d) $\cot^2\alpha - \cot^2\beta - \cot^2\gamma - 1$

22. Let P be a point equidistant from the lines OA, OB and OC where, $\overrightarrow{OA} = 3\hat{i}+4\hat{j}+5\hat{k}$, $\overrightarrow{OB} = 7\hat{i}-\hat{k}$, $\overrightarrow{OC} = 5\hat{i}+5\hat{j}$. Then a vector along $\overrightarrow{OP}$ is

(a) $4\hat{i}+7\hat{j}+9\hat{k}$ (b) $13\hat{i}+4\hat{j}+6\hat{k}$

(c) $9\hat{i}+4\hat{j}+7\hat{k}$ (d) $7\hat{i}+9\hat{j}+4\hat{k}$

SOLUTIONS

1. (c) : The plane passing through $\vec{a}$ and $\vec{b}$ and containing the line $\vec{r} = \vec{b} + \lambda\vec{c}$ is

$$(\vec{r} - \vec{a})\cdot((\vec{b} - \vec{a})\times\vec{c}) = 0$$

$$\Rightarrow \vec{r}\cdot(\vec{b}\times\vec{c} + \vec{c}\times\vec{a}) = [\vec{a}\,\vec{b}\,\vec{c}]$$

Length of ⊥r from the origin $= \dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$

2. (a) : $\vec{c}\cdot\vec{a} = ((\vec{a}\times\vec{c}) + \vec{b})\cdot\vec{a} = \vec{b}\cdot\vec{a}$

$$\vec{b}\times\vec{c} = (\vec{b}\cdot\vec{c})\,\vec{a} - (\vec{b}\cdot\vec{a})\,\vec{c}$$

$$\therefore\ [\vec{a}\,\vec{b}\,\vec{c}] = \vec{b}\cdot\vec{c} - (\vec{a}\cdot\vec{b})\ (\vec{a}\cdot\vec{c}) = \vec{b}\cdot\vec{c} - (\vec{a}\cdot\vec{b})^2$$

Also, $\vec{c}\cdot\vec{b} = 1 - [\vec{a}\,\vec{b}\,\vec{c}]$

$$\therefore\ 2[\vec{a}\,\vec{b}\,\vec{c}] = 1 - (\vec{a}\cdot\vec{b})^2 \le 1$$

$$\therefore\ [\vec{a}\,\vec{b}\,\vec{c}] \le \frac{1}{2}$$

3. (c) : $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d}) = 0$

$$\Rightarrow ([\vec{a}\,\vec{c}\,\vec{d}]\vec{b} - [\vec{b}\,\vec{c}\,\vec{d}]\vec{a})\cdot(\vec{a}\times\vec{d}) = 0$$

$$\Rightarrow [\vec{a}\,\vec{c}\,\vec{d}][\vec{b}\,\vec{a}\,\vec{d}] = 0$$

⇒ either $\vec{c}$ or $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$.

4. (c) : Consider $OABC$, $\overrightarrow{OA} = \vec{\alpha}, \overrightarrow{OB} = \vec{\beta}, \overrightarrow{OC} = \vec{\gamma}$

And OA, BC as a pair of opposite edges.

Given, $|\overrightarrow{OA}| = a, |\overrightarrow{BC}| = b$

Equation of OA is $\vec{r} = \vec{0} + t\vec{\alpha}$

Equation of BC is $\vec{r} = \vec{\beta} + s(\vec{\beta} - \vec{\gamma})$

Now, $d = \dfrac{[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]}{|\vec{\alpha}||\vec{\beta} - \vec{\gamma}|\sin\theta}$, where θ is angle between $\vec{\alpha}$ and $\vec{\beta} - \vec{\gamma}$. $\Rightarrow\ V = \dfrac{abd\sin\theta}{6}$

5. (b) : $\overrightarrow{BC}$ is the x- axis and $\overrightarrow{AB}\perp\overrightarrow{BC}$.

Area of $\Delta ABC = \frac{1}{2}\times\sqrt{2}\times 2 = \sqrt{2}$

Volume $= \frac{1}{3}\times\sqrt{2}\times DE = \frac{2\sqrt{2}}{3} \Rightarrow DE = 2$

6. (c) : $(\vec{a} - \vec{c})\cdot(\vec{b} + \vec{c}) = 0$

$$\Rightarrow \vec{a}\cdot\vec{b} + \vec{c}\cdot(\vec{a} - \vec{b}) - |\vec{c}|^2 = 0$$

$$\Rightarrow (\vec{a} - \vec{b})\cdot\vec{c} = 1 \Rightarrow (\vec{a} - \vec{b})\cdot(\lambda\vec{a} + \mu\vec{b} + \omega(\vec{a}\times\vec{b})) = 1$$

$$\Rightarrow \lambda - \mu = 1 \Rightarrow \lambda = \mu + 1$$

$\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}, \vec{b}, \vec{a}\times\vec{b}$ are mutually perpendicular.

$$|\vec{c}| = 1 \Rightarrow \lambda^2 + \mu^2 + \omega^2 = 1 \Rightarrow (\mu + 1)^2 + \mu^2 + \omega^2 = 1$$

7. (b) : Since $\vec{u}, \vec{m}, \vec{r}$ are mutually perpendicular vectors of same magnitude, we can resolve $\vec{e}$ along the directions $\vec{u}, \vec{m}, \vec{r}$. Let $|\vec{u}| = |\vec{m}| = |\vec{r}| = K$

Also, let $\vec{e} = a\vec{u} + b\vec{m} + c\vec{r}$

Now, $\vec{u}\times\{(\vec{e} - \vec{m})\times\vec{u}\} = (\vec{u}\cdot\vec{u})(\vec{e} - \vec{m}) - (\vec{u}\cdot(\vec{e} - \vec{m}))\vec{u}$

$= K^2(\vec{e} - \vec{m}) - (\vec{u}\cdot\vec{e})\vec{u}$ (∵ $\vec{u}\cdot\vec{m} = 0$)

Similarly, $\vec{m}\times\{(\vec{e} - \vec{r})\times\vec{m}\} = (\vec{m}\cdot\vec{m})(\vec{e} - \vec{r}) - (\vec{m}\cdot(\vec{e} - \vec{r}))\vec{m}$

$= K^2(\vec{e} - \vec{r}) - (\vec{m}\cdot\vec{e})\vec{m}$ (∵ $\vec{m}\cdot\vec{r} = 0$)

And $\vec{r}\times\{(\vec{e} - \vec{u})\times\vec{r}\} = (\vec{r}\cdot\vec{r})(\vec{e} - \vec{u}) - (\vec{r}\cdot(\vec{e} - \vec{u}))\vec{r}$

$= K^2(\vec{e} - \vec{u}) - (\vec{r}\cdot\vec{e})\vec{r}$ (∵ $\vec{r}\cdot\vec{u} = 0$)

Substituting above all, in the given relation, we get

$$\vec{e} = \frac{1}{2}(\vec{u} + \vec{m} + \vec{r})$$

8. (a) : Volume $= \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}] = V$(say)

$$\Rightarrow V^2 = \frac{1}{36}[\vec{a}\,\vec{b}\,\vec{c}]^2 = \frac{1}{36}\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}$$

$$= \frac{1}{36}\left(\frac{3\sqrt{3} - 5}{4}\right)$$

9. (a) : Let the corner meeting the edge and the face be at origin.

Here, $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$(say)

and $\vec{a}\cdot b = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = k^2\cos 60° = \dfrac{k^2}{2}$

(90° – θ) would be the angle between the edge (say $\vec{a}$) and the normal $(\vec{b}\times\vec{c})$ to the face

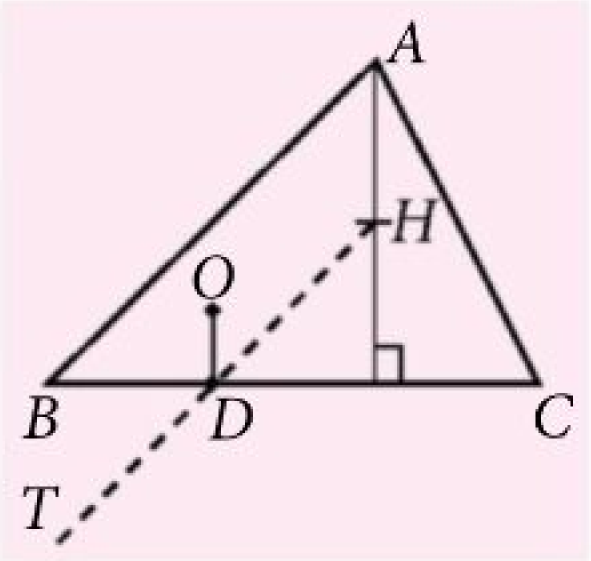
Then, $\sin(90° - \theta) = \dfrac{|\vec{a}\times(\vec{b}\times\vec{c})|}{|\vec{a}||\vec{b}\times\vec{c}|} = \dfrac{|(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}|}{k\cdot k^2\sin 60°}$

$$\Rightarrow \cos\theta = \frac{\frac{k^2}{2}|\vec{b} - \vec{c}|}{k^3\cdot\frac{\sqrt{3}}{2}} = \frac{\frac{k^2}{2}k}{k^3\,\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

10. (c) : Let the circumcentre of ΔABC be origin (O).

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$; and $\overrightarrow{OT} = \vec{t}$

Now, $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OH} \Rightarrow \overrightarrow{OH} = \vec{a} + \vec{b} + \vec{c}$



Also, D is the mid-point of HT.

$\therefore\quad \overrightarrow{AT} = -2\vec{a}$

$\Rightarrow\quad AT = 2R$. But $BC = 2R\sin A = R$ [$\because\ \angle A = 30°$]

$\therefore\quad AT/BC = 2$

11. (d) : Clearly, triangle ABC is an equilateral triangle.

$\Rightarrow\quad l_1 = l_2 = l_3 = \text{in-radius} = \dfrac{1}{\sqrt{6}}$

$\therefore\quad l_1 + l_2 + l_3 = \dfrac{3}{\sqrt{6}} = \dfrac{\sqrt{6}}{2}$

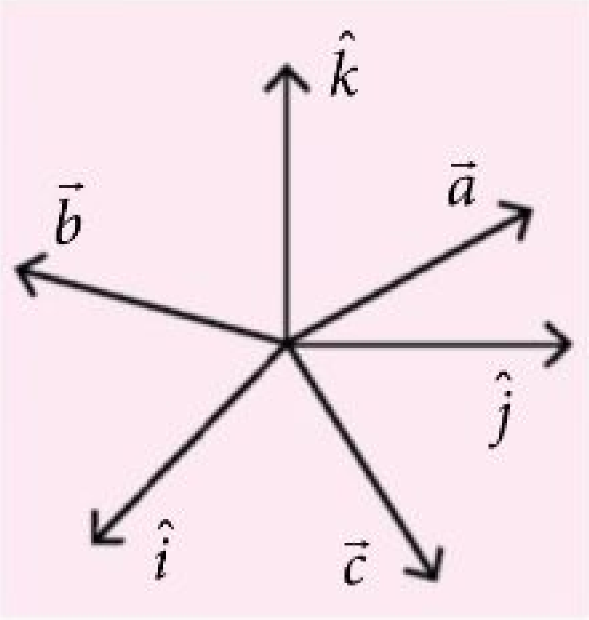
12. (d) : Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = a$ $|\vec{b}| = 2a$ and $|\vec{c}| = 3a$.

$\vec{a} = a\left(\dfrac{\hat{j}+\hat{k}}{\sqrt{2}}\right), \vec{b} = 2a\left(\dfrac{\hat{k}+\hat{i}}{\sqrt{2}}\right),$

$\vec{c} = 3a\left(\dfrac{\hat{j}+\hat{i}}{\sqrt{2}}\right)$

$\therefore\ \vec{a}+\vec{b}+\vec{c} = \dfrac{a}{\sqrt{2}}(5\hat{i}+4\hat{j}+3\hat{k})$

$\Rightarrow\quad |\vec{a}+\vec{b}+\vec{c}| = \dfrac{a}{\sqrt{2}}|5\hat{i}+4\hat{j}+3\hat{k}| = 5a$



13. (a) : Given, $\vec{v} = \vec{a}\times\vec{b} \Rightarrow |\vec{v}| = |\vec{a}||\vec{b}|\sin\theta = \sin\theta$

$\vec{u} = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b} = \vec{a} - \vec{b}\cos\theta$

$\Rightarrow\ |\vec{u}|^2 = (\vec{a}-\vec{b}\cos\theta)^2 = |\vec{a}|^2 + |\vec{b}|^2\cos^2\theta - 2\vec{a}\cdot\vec{b}\cos\theta$

$= 1 + \cos^2\theta - 2\cos^2\theta = 1 - \cos^2\theta = \sin^2\theta$

$\Rightarrow\ |\vec{u}| = |\vec{v}|$

Again $\vec{u}\cdot\vec{b} = \vec{a}\cdot\vec{b} - (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{b}) = 0$

$\Rightarrow\quad \vec{u}\cdot\vec{b} = 0$

14. (a) : $(\lambda-1)(\vec{a}_1 - \vec{a}_2) + \mu(\vec{a}_2 + \vec{a}_3) + \gamma(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2)$
$+ \vec{a}_3 + \delta\vec{a}_4 = \vec{0}$

$\Rightarrow\quad (\lambda-1)\vec{a}_1 + (1-\lambda+\mu-2\gamma)\vec{a}_2 + (\mu+\gamma+1)\vec{a}_3$
$+ (\gamma+\delta)\vec{a}_4 = \vec{0}$

Since $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are linearly independent.

$\therefore\quad \lambda - 1 = 0,\ 1 - \lambda + \mu - 2\gamma = 0,\ \mu + \gamma + 1 = 0,$
$\gamma + \delta = 0$

i.e., $\lambda = 1,\ \mu = 2\gamma,\ \mu + \gamma + 1 = 0,\ \gamma + \delta = 0$

i.e., $\lambda = 1,\ \mu = -\dfrac{2}{3},\ \gamma = -\dfrac{1}{3},\ \delta = \dfrac{1}{3}$

15. (a) : Let $\vec{c} = x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})$

Taking successive dot product with $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}\times\vec{b}$, we get $x = y = \dfrac{1}{3}$ and $z = \dfrac{2\sqrt{2}}{3}$

Also, volume of tetrahedron $OABC = \dfrac{1}{6\sqrt{2}}$.

16. (b) : $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \dfrac{1}{2}$

$\Rightarrow\quad \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \dfrac{1}{\sqrt{2}}$

Now, volume, $V = \dfrac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \dfrac{1}{6\sqrt{2}}$

$\therefore\quad 12\sqrt{2}V = 2$

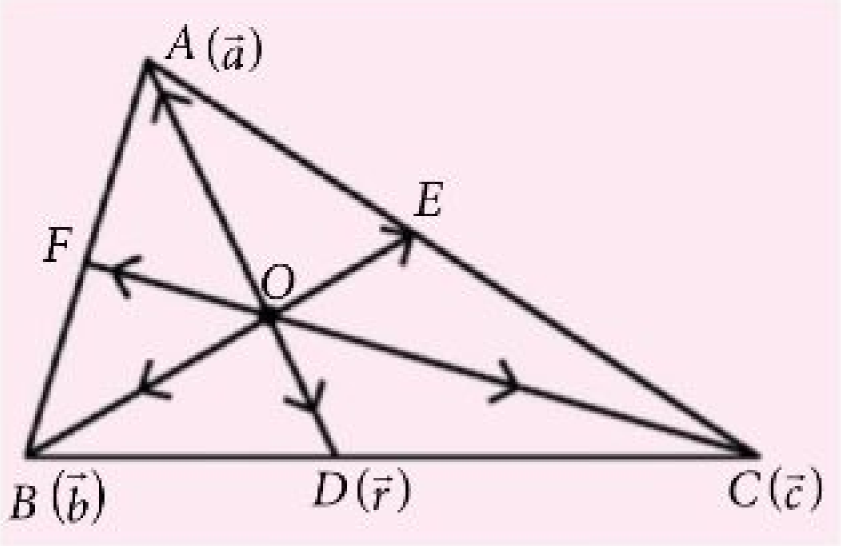
17. (a) : $\overrightarrow{OD} = -x\,\overrightarrow{OA} \ \Rightarrow\ \vec{r} = -x\,\vec{a}$

$\because\quad A, B, C$ are coplanar $\therefore\ l\,\vec{a} + m\vec{b} + n\vec{c} = \vec{0}$

$\Rightarrow\quad l\left(\dfrac{\vec{r}}{-x}\right) + m\vec{b} + n\vec{c} = 0 \ \Rightarrow\ \left(\dfrac{-l}{x}\right)\vec{r} + m\vec{b} + n\vec{c} = 0$

$\because\quad B, C, D$ are collinear.

$\therefore\quad \dfrac{-l}{x} + m + n = 0 \Rightarrow x = \dfrac{l}{m+n}$



$\overrightarrow{AD} = -x\vec{a} - \vec{a} = -(x+1)\vec{a}$

$\therefore\quad \dfrac{OD}{AD} = \dfrac{x}{x+1} = \dfrac{l}{l+m+n}$

Similarly, $\dfrac{OE}{BE} = \dfrac{m}{l+m+n},\ \dfrac{OF}{CF} = \dfrac{n}{l+m+n}$

$\therefore\quad \dfrac{OD}{AD} + \dfrac{OE}{BE} + \dfrac{OF}{CF} = 1$

18. (b) : $\left|\overrightarrow{BC}\right|^2 t^2 - 2(\overrightarrow{BA}\cdot\overrightarrow{BC})t + \left|\overrightarrow{BA}\right|^2 - \left|\overrightarrow{AC}\right|^2 \geq 0$

In standard notations, we have

$t^2a^2 - 2tac\cos B + c^2 - b^2 \geq 0$ ...(i)

$\therefore$ Discriminant of (i) ≤ 0

$\Rightarrow 4a^2c^2\cos^2 B - 4a^2(c^2 - b^2) \leq 0$

$\Rightarrow -c^2\sin^2 B + b^2 \leq 0 \Rightarrow (b - c\sin B)(b + c\sin B) \leq 0$

$\Rightarrow b - c\sin B \leq 0$ $(\because b + c\sin B > 0)$

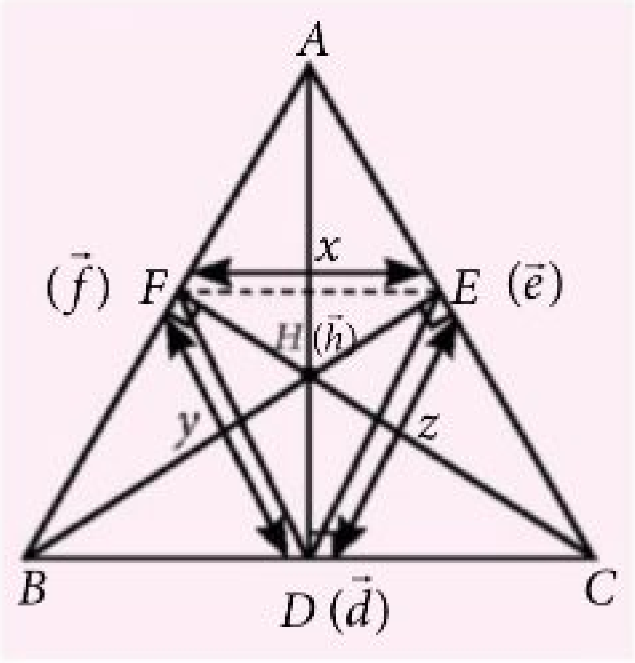
$\Rightarrow b \leq c\sin B \Rightarrow b \leq b\sin C \Rightarrow \sin C = 1 \Rightarrow \angle C = \pi/2$

19. (c) : $\begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix} \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ -a_1 & -a_2 & -a_3 \end{vmatrix} = \left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$

$= |\vec{a}\times\vec{b}|^2|\vec{c}|^2\cos^2\theta$, where θ is the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$

$= |\vec{a}^2||\vec{b}|^2\sin^2\frac{\pi}{6}|\vec{c}|^2\cos^2(0) = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

20. (b) : Note that orthocentre of ΔABC is incentre of ΔDEF. Hence, position vector of H is $\vec{h} = \dfrac{x\vec{d} + y\vec{e} + z\vec{f}}{x+y+z}$



$\Rightarrow \vec{h} - \dfrac{x\vec{d}}{x+y+z} = \dfrac{y\vec{e} + z\vec{f}}{x+y+z}$

Required quantity $= \left|\dfrac{y(\vec{e}-\vec{d}) + z(\vec{f}-\vec{d})}{x+y+z}\right|$

$= \left|\vec{h} - \dfrac{x\vec{d}}{x+y+z} - \dfrac{\vec{d}(y+z)}{x+y+z}\right|$ [From (i)]

$= |\vec{h} - \vec{d}| = |\overrightarrow{HD}| = 2R\cos B\cos C$

$= \dfrac{4(\sqrt{3}+1)(2\sqrt{2})}{\sqrt{3}+1} \times \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}} = 4$

21. (c) : Let $P(x, y, z)$, then $M = (x, 0, z)$, $N = (x, y, 0)$

$\overrightarrow{OM} \times \overrightarrow{ON} = (-yz, zx, xy)$

$$\cos(90° - \theta) = \frac{x(-yz) + y(zx) + z(xy)}{\sqrt{x^2+y^2+z^2}\sqrt{(yz)^2+(zx)^2+(xy)^2}}$$

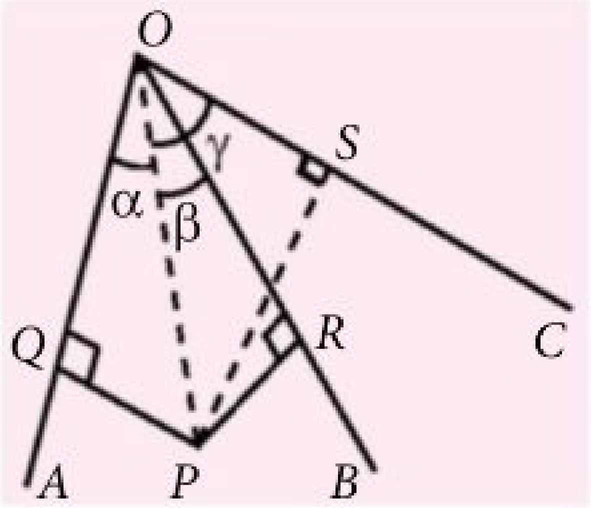
$$\Rightarrow \operatorname{cosec}^2\theta = (x^2+y^2+z^2)\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$$

Also, $\operatorname{cosec}^2\alpha = \dfrac{x^2+y^2+z^2}{x^2}$, $\operatorname{cosec}^2\beta = \dfrac{x^2+y^2+z^2}{y^2}$

and $\operatorname{cosec}^2\gamma = \dfrac{x^2+y^2+z^2}{z^2}$

$\therefore \operatorname{cosec}^2\theta = \operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma$

22. (b) : Let $\angle AOP = \alpha$, $\angle BOP = \beta$, $\angle COP = \gamma$



Given, $PQ = PR = PS$

$\Rightarrow OP\sin\alpha = OP\sin\beta = OP\sin\gamma$

$\Rightarrow \alpha = \beta = \gamma$

Also, $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = \left|\overrightarrow{OC}\right|$

$\therefore \overrightarrow{OP}\cdot\overrightarrow{OA} = \overrightarrow{OP}\cdot\overrightarrow{OB} = \overrightarrow{OP}\cdot\overrightarrow{OC}$

$\Rightarrow \overrightarrow{OP} \perp (\overrightarrow{OA} - \overrightarrow{OB})$ and $\overrightarrow{OP} \perp (\overrightarrow{OA} - \overrightarrow{OC})$

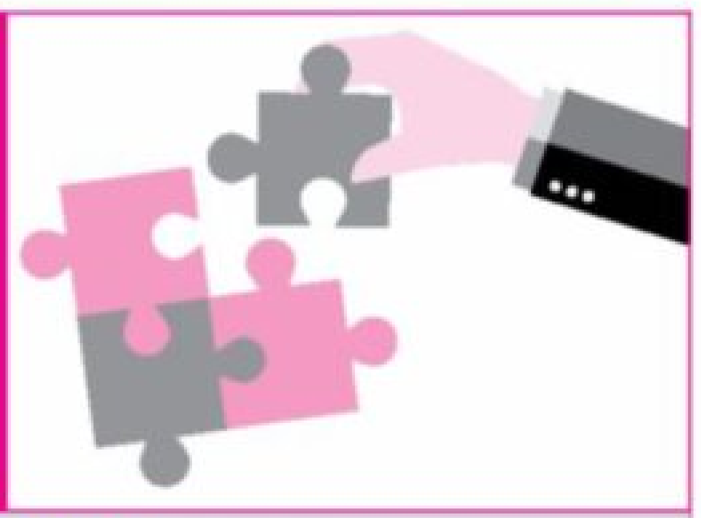
$\Rightarrow \overrightarrow{OP} \parallel (\overrightarrow{OA} - \overrightarrow{OB}) \times (\overrightarrow{OA} - \overrightarrow{OC})$

$\therefore$ Required vector along $\overrightarrow{OP}$

$= 2(13\hat{i} + 4\hat{j} + 6\hat{k})$

◆◆

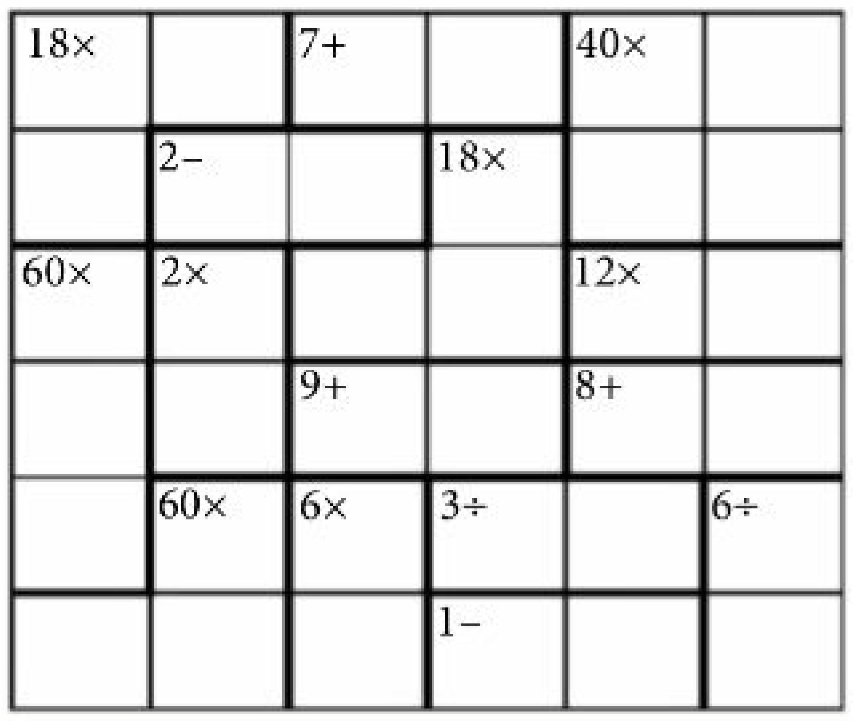
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



18×		7+		40×	
	2–		18×		
60×	2×			12×	
		9+		8+	
	60×	6×	3÷		6÷
			1–		

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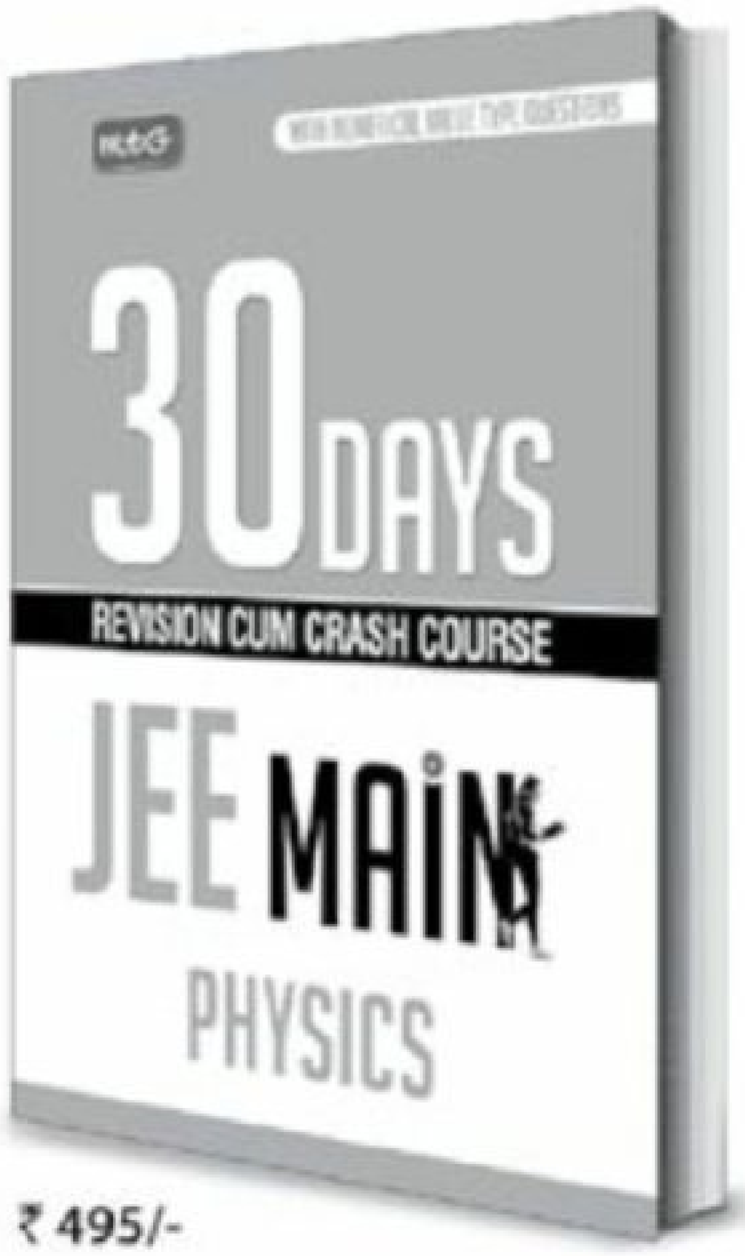
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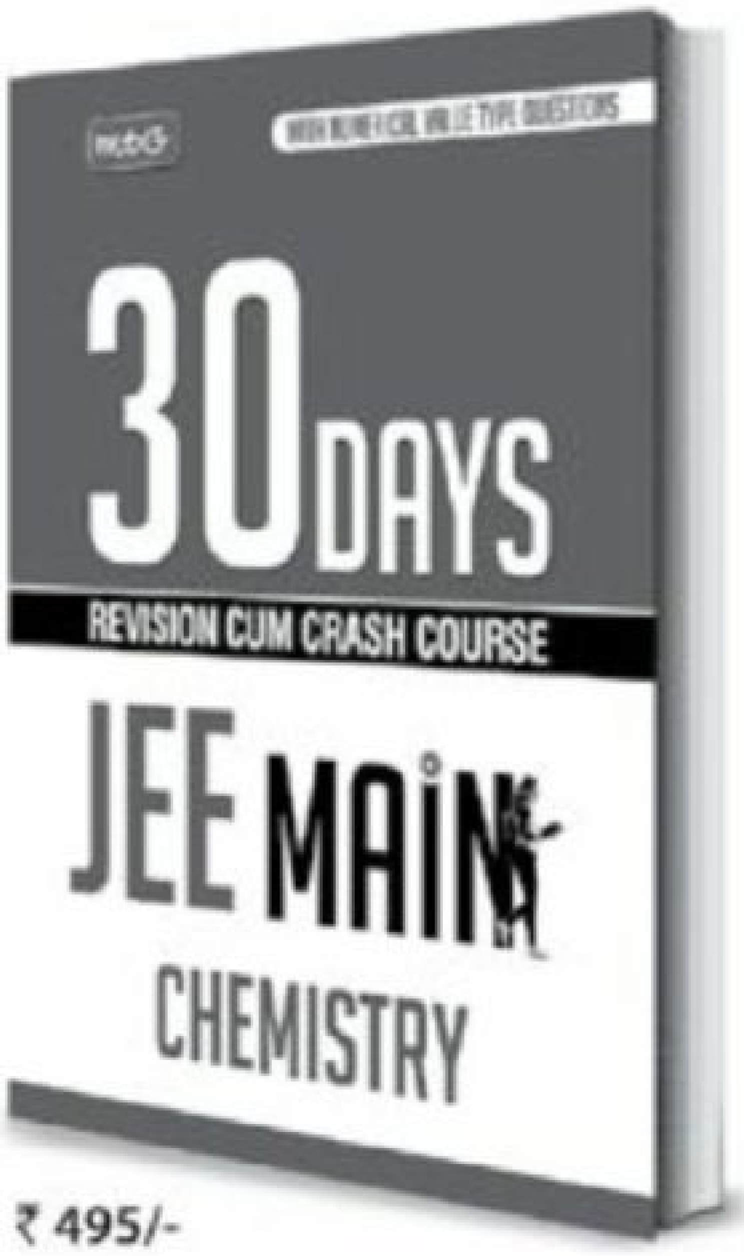
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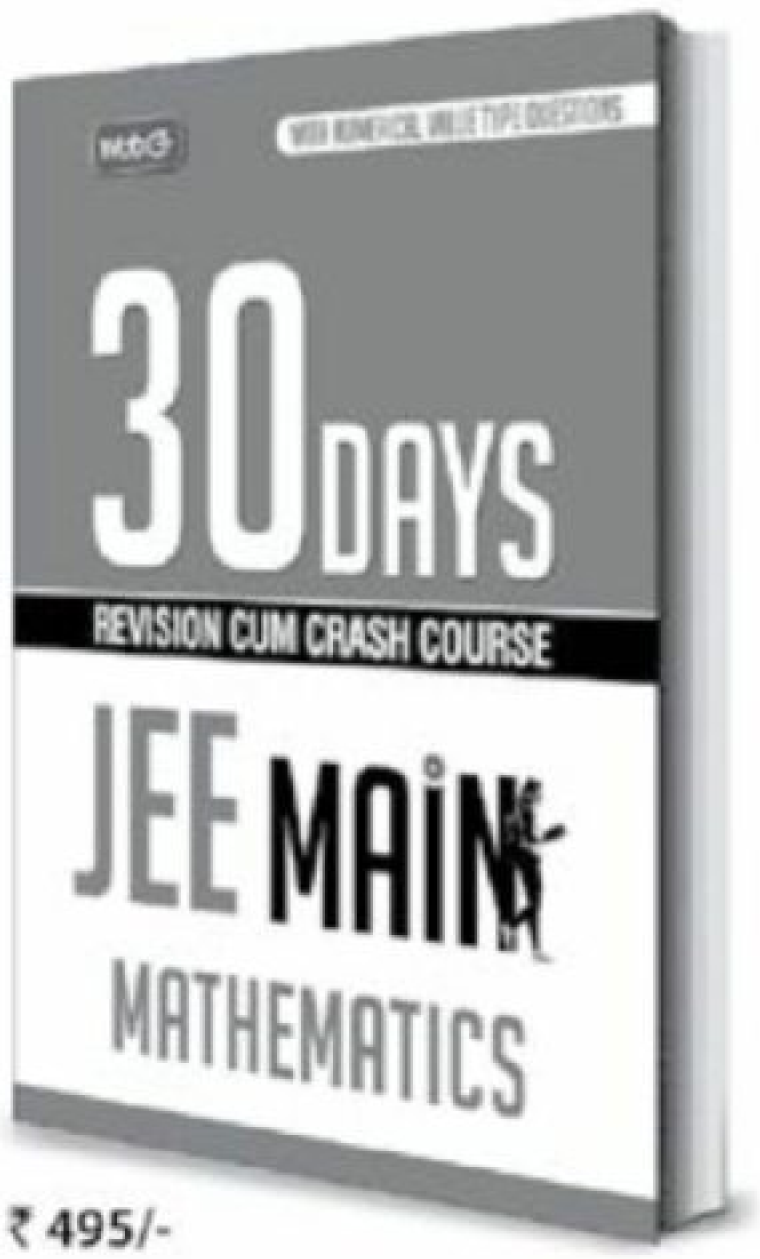
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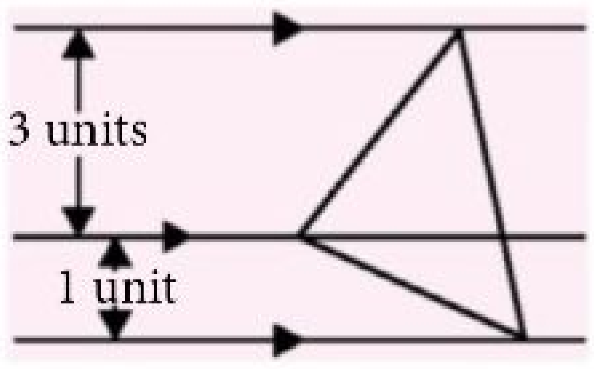
OLYMPIAD CORNER

1. The vertices of an equilateral triangle lie on three parallel lines which are 3 units and 1 unit apart as shown. Find the area of this triangle.



2. Draw a tangent line to parabola $y = x^2$ at the point $A(1, 1)$. Suppose the line intersects the x-axis and y-axis at D and B respectively. Let point C be on the parabola and point E on AC such that $\frac{AE}{EC} = \lambda_1$. Let point F be on BC such that $\frac{BF}{FC} = \lambda_2$ and $\lambda_1 + \lambda_2 = 1$. Assume that CD intersects EF at point P. When point C moves along the parabola, find the equation of the trail of P.

3. Find the largest constant k such that
$$\frac{kabc}{a+b+c} \le (a+b)^2 + (a+b+4c)^2 \text{ for all } a, b, c, > 0.$$

4. Let n be a positive integer and a a positive real number. Determine the maximum value of $a^{k(1)} + a^{k(2)} + ... + a^{k(s)}$, where s is an integer such that $1 \le s \le n$ and $k(1), k(2), ..., k(s)$ are positive integers with sum n.

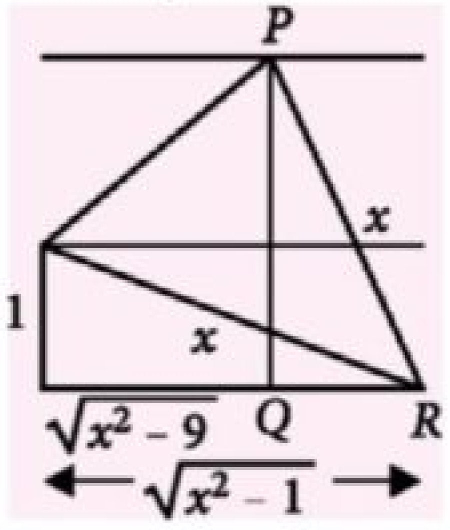
5. Let a_0 and a_1 be integers. The sequence a_n is defined by $a_2 = 2a_1 - a_0 + 2$ and $a_{n+1} = 3a_n - 3a_{n-1} + a_{n-2}$ for $n \ge 2$. If for any positive integer m, the sequence contains m consecutive terms all of which are perfect squares, prove that every term of the sequence is a perfect square.

6. Find the value of the continued root:
$$\sqrt{4+27\sqrt{4+29\sqrt{4+31\sqrt{4+33\sqrt{\cdots}}}}}.$$

7. Prove that the average of the numbers $n \sin n°$, $n = 2, 4, 6, ..., 180$ is $\cot 1°$.

SOLUTIONS

1. Using Pythagoras' Theorem on the triangle marked PQR, we have



$$4^2 + \left(\sqrt{x^2-1} - \sqrt{x^2-9}\right)^2 = x^2$$
$$\Rightarrow\ 16 + x^2 - 1 + x^2 - 9 - 2\sqrt{\left(x^2-1\right)\left(x^2-9\right)} = x^2$$

On rearrangement, we get

$$x^2 + 6 = 2\sqrt{\left(x^2-1\right)\left(x^2-9\right)} \qquad ...(i)$$

Squaring (i) both sides, we get

$$x^4 + 12x^2 + 36 = 4(x^4 - 10x^2 + 9) = 4x^4 - 40x^2 + 36$$

This simplifies to $3x^4 - 52x^2 = 0$ or $x^2 = 52/3$. The area of an equilateral triangle of side length x is half base times height, *i.e.*, $1/2 \times x \times \sqrt{3}x/2$ or $x^2 \times \sqrt{3}/4$.

The area of the given triangle is thus $52\sqrt{3}/12 = 13\sqrt{3}/3$.

2. The slope of the tangent line passing through A is $y' = 2x|_{x=1} = 2$. So the equation of the tangent line AB is $y = 2x - 1$.

Hence, the coordinates of B and D are $B(0, -1)$, $D\left(\frac{1}{2}, 0\right)$.

Thus, D is the midpoint of line segment AB.

Consider $P(x, y)$, $C(x_0, x_0^2)$, $E(x_1, y_1)$, $F(x_2, y_2)$.

Then by $\frac{AE}{EC} = \lambda_1$, we know $x_1 = \frac{1+\lambda_1 x_0}{1+\lambda_1}$, $y_1 = \frac{1+\lambda_1 x_0^2}{1+\lambda_1}$. From $\frac{BF}{FC} = \lambda_2$, we get $x_2 = \frac{\lambda_2 x_0}{1+\lambda_2}$, $y_2 = \frac{-1+\lambda_2 x_0^2}{1+\lambda_2}$.

Therefore the equation of line EF is

$$\frac{y - \frac{1+\lambda_1 x_0^2}{1+\lambda_1}}{\frac{-1+\lambda_2 x_0^2}{1+\lambda_2} - \frac{1+\lambda_1 x_0^2}{1+\lambda_1}} = \frac{x - \frac{1+\lambda_1 x_0}{1+\lambda_1}}{\frac{\lambda_2 x_0}{1+\lambda_2} - \frac{1+\lambda_1 x_0}{1+\lambda_1}}.$$

Simplifying it, we get

$$[(\lambda_2 - \lambda_1)x_0 - (1 + \lambda_2)]y$$

$$= [(\lambda_2 - \lambda_1)\, x_0^2 - 3]\, x + 1 + x_0 - \lambda_2 x_0^2. \quad \text{...(i)}$$

When $x_0 \neq \frac{1}{2}$, the equation of line CD is

$$y = \frac{2x_0^2 x - x_0^2}{2x_0 - 1} \quad \text{...(ii)}$$

From (i) and (ii), we get $\begin{cases} x = \dfrac{x_0 + 1}{3}, \\ y = \dfrac{x_0}{3}. \end{cases}$

Eliminating x_0, we get the equation of the trail of point P as $y = \frac{1}{3}(3x-1)^2$.

When $x_0 = \frac{1}{2}$, the equation of EF is

$-\frac{3}{2}y = \left(\frac{1}{4}\lambda_2 - \frac{1}{4}\lambda_1 - 3\right)x + \frac{3}{2} - \frac{1}{4}\lambda_2$, the equation of CD is $x = \frac{1}{2}$. Combining them, we conclude that $(x, y) = \left(\frac{1}{2}, \frac{1}{12}\right)$ is on the trail of P. Since C and A cannot be congruent, $x_0 \neq 1, x \neq \frac{2}{3}$.

Therefore the equation of the trail is

$$y = \frac{1}{3}(3x-1)^2,\ x \neq \frac{2}{3}.$$

3. By the A.M.-G.M. inequality,

$$(a + b)^2 + (a + b + 4c)^2 = (a + b)^2 + (a + 2c + b + 2c)^2$$

$$\geq (2\sqrt{ab})^2 + (2\sqrt{2ac} + 2\sqrt{2bc})^2$$

$$= 4ab + 8ac + 8bc + 16c\sqrt{ab}.$$

Therefore,

$$\frac{(a+b)^2 + (a+b+4c)^2}{abc} \cdot (a+b+c)$$

$$\geq \frac{4ab + 8ac + 8bc + 16c\sqrt{ab}}{abc} \cdot (a+b+c)$$

$$= \left(\frac{4}{c} + \frac{8}{b} + \frac{8}{a} + \frac{16}{\sqrt{ab}}\right)(a+b+c)$$

$$= 8\left(\frac{1}{2c} + \frac{1}{b} + \frac{1}{a} + \frac{1}{\sqrt{ab}} + \frac{1}{\sqrt{ab}}\right)\left(\frac{a}{2} + \frac{a}{2} + \frac{b}{2} + \frac{b}{2} + c\right)$$

$$\geq 8\left(5\sqrt[5]{\frac{1}{2a^2b^2c}}\right)\left(5\sqrt[5]{\frac{a^2b^2c}{2^4}}\right) = 100,$$

again by the A.M.-G.M. Inequality. Hence the largest constant k is 100. For $k = 100$, equality holds if and only if $a = b = 2c > 0$.

4. If $0 < a \leq 1$, a^l is non-increasing as the positive integer l increases. Hence we should take $s = n$ and $k(i) = 1$ for $1 \leq i \leq s$. The maximum value of $a^{k(1)} + a^{k(2)} + ... + a^{k(s)}$ ...(i)

is na. Now let $a > 1$. For all positive integers u and v,

$$a(a^{u-1} - 1)(a^{v-1} - 1) \geq 0,$$

or $a^u + a^v \leq a + a^{u+v-1}$

Hence we should take $k(i) = 1$ for $1 \leq i \leq s - 1$. The maximum value of (i) is then at most

$$(s - 1)\, a + a^{n-(s-1)} \quad \text{...(ii)}$$

Note that $a + a^m = a^{m+1}$ if

$$m = \log_a \frac{a}{a-1} \quad \text{...(iii)}$$

It follows that if

$$s \leq (n + 1) - \log_a \frac{a}{a-1}, \quad \text{...(iv)}$$

COMIC CAPSULE

Mr. Hypotenuse, I'll need you to **sine** this, and Mrs., I'll need you to **cosine**. I'll need to stop at the store on the way to home for milk and bread and then ...
This guy always goes off on a **tangent**.
LEASE

The value of (ii) decreases as s decreases, so that we should take $s = n$ and obtain na. On the other hand, if

$$s > (n + 1) - \log_a \frac{a}{a-1}, \qquad ...(v)$$

the value of (ii) decreases as s increases, so that we should take $s = 1$ and obtain a^n. Hence, the maximum value of (ii), and of (v), is $\max\{na, a^n\}$. For $n = 1$, $na = a^n$. Suppose $n \geq 2$. Then $na = a^n$ when $a = n^{\frac{1}{n-1}}$.

Hence, if $a \leq n^{\frac{1}{n-1}}$, the maximum value of (i) is na. If $a > n^{\frac{1}{n-1}}$, the maximum value of (i) is a^n.

5. Let $d_n = a_n - a_{n-1}$ for $n \geq 1$.

Then, $0 = a_{n+1} - 3a_n + 3a_{n-1} - a_{n-2} = d_{n+1} - 2d_n + d_{n-1}$

For $n \geq 2$. Hence

$$d_n - d_{n-1} = d_{n-1} - d_{n-2} = \ldots = d_2 - d_1 = a_2 - 2a_1 + a_0 = 2$$

For $n \geq 1$, so that

$$a_n = a_0 + \sum_{k=1}^{n} d_k = a_0 + nd_1 + n(n-1)$$

$$= n^2 + (a_1 - a_0 - 1)n + a_0$$

By the hypothesis, there exists a positive integer t such that a_t and a_{t+2} are both squares. Hence $a_{t+2} - a_t \not\equiv 2 (\text{mod } 4)$.

Since, $a_{t+2} - a_t = 4t + 4 + 2(a_1 - a_0 - 1)$,

we must have $a_1 - a_0 - 1 = 2\lambda$ for some integer λ and $a_n = (n + \lambda)^2 + a_0 - \lambda^2$.

For $n \geq 0$. If $a_0 - \lambda^2 \neq 0$, let it have m divisors. Since the coefficient of the quadratic term in a_n is positive, there exists a positive integer n_0 such that $\{a_n\}$ is strictly increasing for $n > n_0$. By the hypothesis, there exists a positive integer $k > n_0$ such that for $1 \leq i \leq m$, $a_{k+i} = b_i^2$ for some positive integer b_i.

Then, $a_0 - \lambda^2 = b_i^2 - (k + i + \lambda)^2 = (b_i - k - i - \lambda)(b_i + k + i + \lambda)$

For $1 \leq i \leq m$, the integers $b_i + k + i\lambda$ are distinct since $b_i + i < b_{i+1} + i + 1$. Thus $a_0 - \lambda^2$ has at least $m + 1$ divisors, which is a contradiction. It follows that $a_0 - \lambda^2 = 0$ and $a_n = (n + \lambda)^2$ for $n \geq 0$.

6. The answer is 29. More generally, for any positive integer n, we claim that

$$\sqrt{4 + n\sqrt{4 + (n+2)\sqrt{4 + (n+4)\sqrt{\cdots}}}} = n + 2,$$

where the left side is defined as the limit of

$$F(n, m) = \sqrt{4 + n\sqrt{4 + (n+2)\sqrt{4 + (n+4)\sqrt{\ldots\sqrt{4 + m\sqrt{4}}}}}}$$

as $m \to \infty$ (where m is an integer and $(m - n)$ is even)

If $g(n, m) = F(n, m) - (n + 2)$, we have

$$F(n, m)^2 - (n+2)^2 = (4 + nF(n+2, m)) - (4 + n(n+4))$$

$$= n(F(n + 2, m) - (n + 4))$$

so $g(n, m) = \dfrac{n}{F(n, m) + n + 2} g(n + 2, m)$.

Clearly $F(n, m) > 2$, so

$$|g(n, m)| < \frac{n}{n+4} |g(n + 2, m)|.$$

By iterating this, we obtain

$$|g(n, m)| < \frac{n(n+2)}{m(m+2)} |g(m, m)| < \frac{n(n+2)}{m}$$

Therefore $g(n, m) \to 0$ as $m \to \infty$.

7. Let $x = 2\sin 2° + 4\sin 4° + \ldots + 90\sin 90° + \ldots + 178\sin 178°$

$= (2 + 178)\sin 2° + (4 + 176)\sin 4° + \ldots$

$= 180(\sin 2° + \sin 4° + \ldots + \sin 88°) + 90\sin 90°$

Then, $\bar{x} = \dfrac{x}{90} = 2\sin 2° + 2\sin 4° + \ldots + 2\sin 88° + 1$

$\Rightarrow \bar{x}\ \sin 1° = 2\sin 2° \sin 1° + 2\sin 4° \sin 1° + \ldots + 2\sin 88° \sin 1° + \sin 1°.$

Now, $2\sin 2° \sin 1° = \cos 1° - \cos 3°$,

$2\sin 4° \sin 1° = \cos 3° - \cos 5°$,

....

$2\sin 88° \sin 1° = \cos 87° - \cos 89°$

Hence, $\bar{x}\sin 1° = \cos 1° - \cos 89° + \sin 1° = \cos 1°$.

Thus, $\bar{x} = \cot 1°$, as required.

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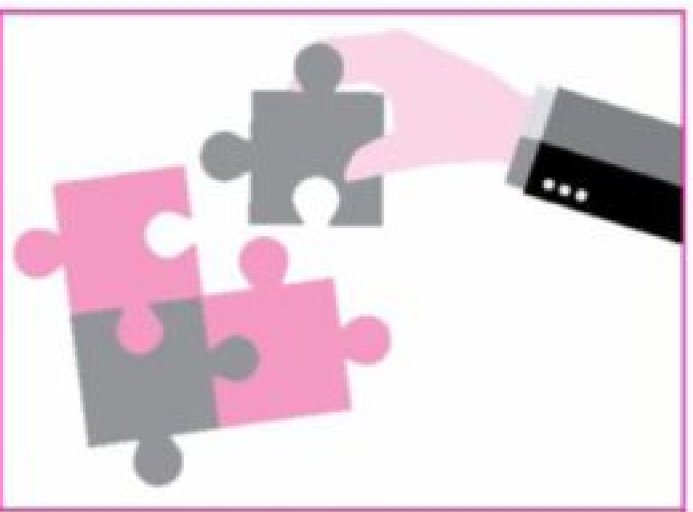
PUZZLE CORNER

ANSWER - NOVEMBER 2019



3– 6	3÷ 1	3	11+ 5	2	4
3	40× 5	10+ 4	2÷ 2	6÷ 6	1
2× 1	2	6	4	15× 5	3
2	4	6+ 5	3÷ 1	3	3÷ 6
15+ 5	6	1	19+ 3	4	2
4	6× 3	2	6	1	5

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Prove that the sum of infinite natural numbers is $-\frac{1}{12}$.

(Mukul Prajapat, Rajasthan)

Ans. Let $S = 1 + 2 + 3 + 4 + \ldots\ldots \infty$ ....(i)

and $S_1 = 1 - 1 + 1 - 1 + \ldots\ldots \infty$ ....(ii)

Now, this sum should be 0 or 1 based on number of natural numbers taken. If infinite numbers are even, $S_1 = 0$, if odd, $S_1 = 1$. But Riemann zeta function gives it a value of $\frac{1}{2}$.

$\because \quad S_1 = 1 - 1 + 1 - 1 + \ldots\ldots \infty$

$\therefore \quad 1 - S_1 = 1 - (1 - 1 + 1 - 1 + \ldots\ldots \infty)$

$\Rightarrow \quad 1 - S_1 = 1 - 1 + 1 - 1 + \ldots\ldots \infty$

$\Rightarrow \quad 1 - S_1 = S_1 \Rightarrow S_1 = \frac{1}{2}$ ....(iii)

Now, let $S_2 = 1 - 2 + 3 - 4 + 5 - 6 + \ldots\ldots \infty$ ....(iv)

So, $\quad S_2 = \quad 1 - 2 + 3 - 4 + 5 - \ldots\ldots \infty$ ....(v)

Adding (iv) and (v), we get

$2S_2 = 1 - 1 + 1 - 1 + 1 - 1 + \ldots\ldots \infty$

$\Rightarrow \quad 2S_2 = S_1 \quad \Rightarrow \quad 2S_2 = \frac{1}{2}$ [From (iii)]

$\Rightarrow \quad S_2 = \frac{1}{4}$ ....(vi)

Again from (i) and (iv),

$S - S_2 = (1 + 2 + 3 + 4 + \ldots\ldots \infty) - (1 - 2 + 3 - 4 + 5 - 6 + \ldots\ldots \infty)$

$\Rightarrow \quad S - S_2 = 4 + 8 + 12 + 16 + \ldots\ldots \infty$

$\Rightarrow \quad S - S_2 = 4\,(1 + 2 + 3 + 4 + \ldots\ldots \infty)$

$\Rightarrow \quad S - S_2 = 4S$ [From (i)]

$\Rightarrow \quad 3S = -S_2 \Rightarrow S = -\frac{1}{3}S_2 = -\frac{1}{12}$ [From (vi)]

It is interesting to know $S = -1/12$ has been used to derive the equations in string theory, quantum field theory and in some complex analytics.

2. If $u = \sqrt{z_1 z_2}$, prove that

$$|z_1| + |z_2| = \left|\frac{z_1 + z_2}{2} + u\right| + \left|\frac{z_1 + z_2}{2} - u\right|.$$

(Anchal Srivastava, M.P.)

Ans. $\text{R.H.S.} = \left|\frac{z_1 + z_2}{2} + \sqrt{z_1 z_2}\right| + \left|\frac{z_1 + z_2}{2} - \sqrt{z_1 z_2}\right|$

$= \frac{1}{2}\left|z_1 + z_2 + 2\sqrt{z_1 z_2}\right| + \frac{1}{2}\left|z_1 + z_2 - 2\sqrt{z_1 z_2}\right|$

$= \frac{1}{2}\left|(\sqrt{z_1} + \sqrt{z_2})^2\right| + \frac{1}{2}\left|(\sqrt{z_1} - \sqrt{z_2})^2\right|$

$= \frac{1}{2}\left|\sqrt{z_1} + \sqrt{z_2}\right|^2 + \frac{1}{2}\left|\sqrt{z_1} - \sqrt{z_2}\right|^2$

$[\because |z^2| = |z \cdot z| = |z|\,|z| = |z|^2]$

Now, for any two complex numbers α, β, we get

$|\alpha + \beta|^2 + |\alpha - \beta|^2$

$= (\alpha + \beta)\overline{(\alpha + \beta)} + (\alpha - \beta)\overline{(\alpha - \beta)}$ $\quad [\because |z|^2 = z\bar{z}]$

$= (\alpha + \beta)(\bar{\alpha} + \bar{\beta}) + (\alpha - \beta)(\bar{\alpha} - \bar{\beta})$

$[\because \overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2]$

$= \alpha\bar{\alpha} + \alpha\bar{\beta} + \beta\bar{\alpha} + \beta\bar{\beta} + \alpha\bar{\alpha} - \alpha\bar{\beta} - \beta\bar{\alpha} + \beta\bar{\beta}$

$= 2|\alpha|^2 + 2|\beta|^2$

$\therefore \quad \frac{1}{2}|\alpha + \beta|^2 + \frac{1}{2}|\alpha - \beta|^2 = |\alpha|^2 + |\beta|^2$

So, $\text{R.H.S.} = |\sqrt{z_1}|^2 + |\sqrt{z_2}|^2 = |(\sqrt{z_1})^2| + |(\sqrt{z_2})^2|$

$= |z_1| + |z_2| = \text{L.H.S.}$

3. A person has to go through three successive tests. The probability of passing first test is p. If he fails in one of the tests, then the probability of passing next test is $p/2$, otherwise it remains the same. For selection, the person must pass atleast two tests. Find the probability that the person has to be selected.

(Abhijit Pal, W.B.)

Ans. Let E_i $(i = 1, 2, 3)$ be the event of the person passing the i^{th} test and E be the event that he is selected.

Then $E = (E_1 \cap E_2) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3)$

Therefore

$$P(E) = P(E_1)P(E_2\,|\,E_1) + P(E_1)P(\bar{E}_2\,|\,E_1)P(E_3\,|\,\bar{E}_2) + P(\bar{E}_1)P(E_2\,|\,\bar{E}_1)P(E_3\,|\,E_2)$$

$= p \cdot p + p(1 - p)\frac{p}{2} + (1 - p)\frac{p}{2} \cdot p$

$= 2p^2 - p^3$

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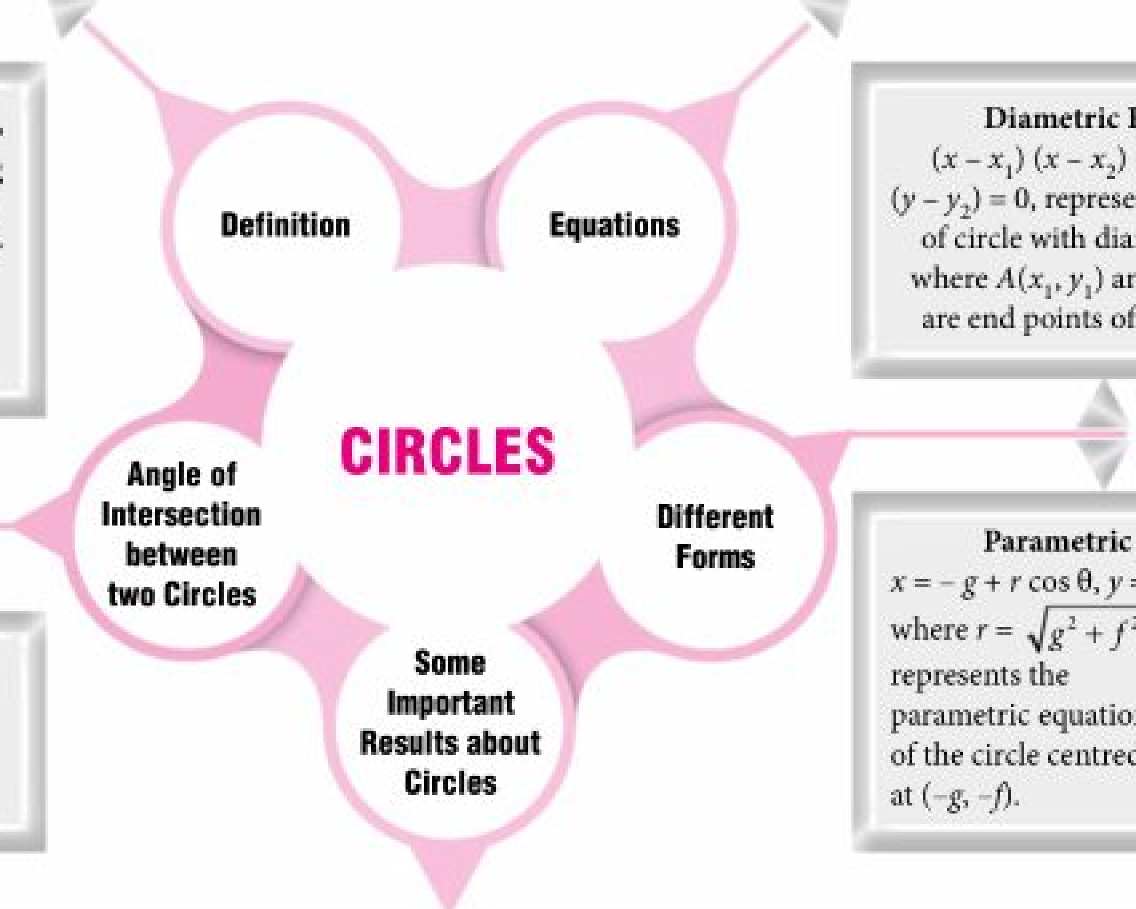
CONCEPT MAP

CIRCLES

Class XI

A circle is defined as the locus of a point, which moves in a plane such that its distance from a fixed point in that plane is always constant. The fixed point is called the centre and the constant distance is called the radius of the circle.

- General equation of circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$ with centre $(C) = (-g, -f)$ and radius $(r) = \sqrt{g^2 + f^2 - c}$.
- Standard equation of circle is given by $(x - h)^2 + (y - k)^2 = a^2$ with centre $(C) = (h, k)$ and radius $(r) = a$.
- The second degree equation given by $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, represents a circle if $a = b$, $h = 0$, $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ and $g^2 + f^2 - ac > 0$.


Definition
Equations
CIRCLES
Angle of Intersection between two Circles
Different Forms
Some Important Results about Circles

The angle between two intersecting circles having centres at C_1 & C_2 and radii r_1 & r_2 respectively is given by $\cos\theta = \frac{r_1^2 + r_2^2 - (C_1C_2)^2}{2r_1r_2}$.

Diametric Form

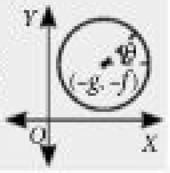
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, represents equation of circle with diameter AB, where $A(x_1, y_1)$ and $B(x_2, y_2)$ are end points of diameter.

Orthogonal Circles

Two circles are orthogonal if $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

Parametric Form

$x = -g + r\cos\theta$, $y = -f + r\sin\theta$ where $r = \sqrt{g^2 + f^2 - c}$ represents the parametric equation of the circle centred at $(-g, -f)$.



- Let $S = x^2 + y^2 + 2gx + 2fy + c$, $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ and $T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$
  - Equation of tangent to the circle $S = 0$ at (x_1, y_1) is given by $T = 0$.
  - Equation of chord of the circle $S = 0$ whose mid-point is (x_1, y_1) is $T = S_1$.
  - Length of intercept made by the circle $S = 0$ with x-axis $= 2\sqrt{g^2 - c}$.
  - Length of intercept made by the circle $S = 0$ with y-axis $= 2\sqrt{f^2 - c}$.
  - Length of tangent from a point (x_1, y_1) to the circle $S = 0$ is given by $\sqrt{S_1}$.
  - Joint equation of pair of tangents drawn from the point (x_1, y_1) to the circle $S = 0$ is given by $T^2 = SS_1$.
- Equation of radical axis of two circles $S_1 = x^2 + y^2 + 2gx + 2fy + c = 0$ and $S_2 = x^2 + y^2 + 2g'x + 2f'y + c' = 0$ is given by $S_1 - S_2 = 0$.
- Line $y = mx + c$ will touch the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.
- Equation of director circle of the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.

LINE

Class XII

CONCEPT MAP

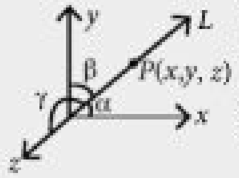
Direction Cosines and Direction Ratios

- **Direction cosines :** If a line L makes an angle α, β, γ $(0 \le \alpha, \beta, \gamma \le \pi)$ with the positive direction of x, y, z axes respectively then $(l, m, n) = (\cos\alpha, \cos\beta, \cos\gamma)$ are the direction cosines of the line L and $l^2 + m^2 + n^2 = 1$.

$$\cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\beta = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$



D.C.s of axes:
x-axis : (1, 0, 0)
y-axis : (0, 1, 0)
z-axis : (0, 0, 1)

- **Direction ratios :** Any three numbers proportional to (l, m, n) are called direction ratios. We denote them by (a, b, c).

$$\therefore \quad \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k$$

Shortest Distance between Two Lines

- **Vector form :** Let l_1 and l_2 be two skew lines whose equations are $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ respectively. Let $\overrightarrow{PQ}$ be the shortest distance vector between l_1 and l_2. Then, $PQ = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$
  If lines are parallel, then
  $\vec{b}_1 = \vec{b}_2 = \vec{b}$ and $d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$.
- **Cartesian form :** Let two skew lines be
  $l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and
  $l_2 : \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$.
  The shortest distance between l_1 and l_2 is given by

$$\left|\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}\right|.$$

Equation of Line in Different Forms

- **Equation of a line in space :**
  Vector form : $\vec{r} = \vec{a} + \lambda\vec{b}$, $\lambda \in R$, where $A(\vec{a})$ is passing point and $\vec{b}$ is the parallel vector.
  Cartesian form : $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$, where (x_1, y_1, z_1) is passing point and (a_1, b_1, c_1) are direction ratios of the line.
- **Line passing through two points :**
  Vector form : $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, $\lambda \in R$, where $A(\vec{a})$ and $B(\vec{b})$ are the passing points.
  Cartesian form : $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$, where $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are the passing points.

Angle Between Two Lines

- **Vector form :** Let $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$ & $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$ be the two lines then $\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$.
- **Cartesian form :** Let $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ be the two lines then
  $\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$.

Coplanarity of Two Lines

- **Vector form :** Two lines $\vec{r}_1 = a_1 + \lambda_1\vec{n}_1$, $\vec{r}_2 = \vec{a}_2 + \lambda_2\vec{n}_2$ are coplanar if $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{n}_1 \times \vec{n}_2) = 0$.
- **Cartesian form :** Two lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0.$$

MATHS MUSING

SOLUTION SET-203

1. (d) : 1 to 9 ⇒ 9 digits, 10 to 99 ⇒ 180 digits
100 to 699 ⇒ 1800 digits ∴ 1 to 699 ⇒ 1989 digits
Now, 700, 701, 702, 703, 704, 705, 706, 707, 708
2014th digit = (1989 + 8 × 3 + 1)th digit = 7, the first digit of 708.

2. (a) : $7^{2014} = -(1-50)^{1007}$

$= -[1 - 50350 + 1007\cdot503\cdot2500 - 1007\cdot503\cdot335\cdot125000 + \dots]$

∴ Required last four digits

$= -1 + 350 - 2500 + 5000 = 2849$, digit sum = 23.

3. (c) : $\left(\frac{a+b+c+d}{4}\right)^2 \le \frac{a^2+b^2+c^2+d^2}{4}$

$\Rightarrow (8-p)^2 \le 4(16-p^2) \Rightarrow p \le \frac{16}{5}$

4. (d) : $I = \int_0^{\sqrt{e-1}} 1\cdot dx + \int_{\sqrt{e-1}}^{2} \ln(1+x^2)\,dx$

$= \sqrt{e-1} + x\ln(1+x^2)\Big|_{\sqrt{e-1}}^{2} - 2\int_{\sqrt{e-1}}^{2}\left(1-\frac{1}{1+x^2}\right)dx$

$= \sqrt{e-1} + 2\ln 5 - \sqrt{e-1} - 2\left[x - \tan^{-1}x\right]_{\sqrt{e-1}}^{2}$

$= 2\ln 5 - 4 + 2\sqrt{e-1} + 2\tan^{-1}2 - 2\tan^{-1}\sqrt{e-1}$

⇒ Required answer = –4

5. (b) : Required probability

$= {}^7C_3\,4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)\div 7! = \frac{1}{16}$

6. (a, d) : Bisector of $\angle BAC$ is $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} \pm \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$

⇒ Direction ratios are –1, 2, 1 and 2, –1, 4

7. (c) : $f - xf' = g - xg' \Rightarrow f - g = cx$

$x + ff' = x + gg' \Rightarrow f^2 - g^2 = d$

$\therefore\ f + g = \frac{d}{cx},\ f = \frac{1}{2}\left(cx + \frac{d}{cx}\right)$

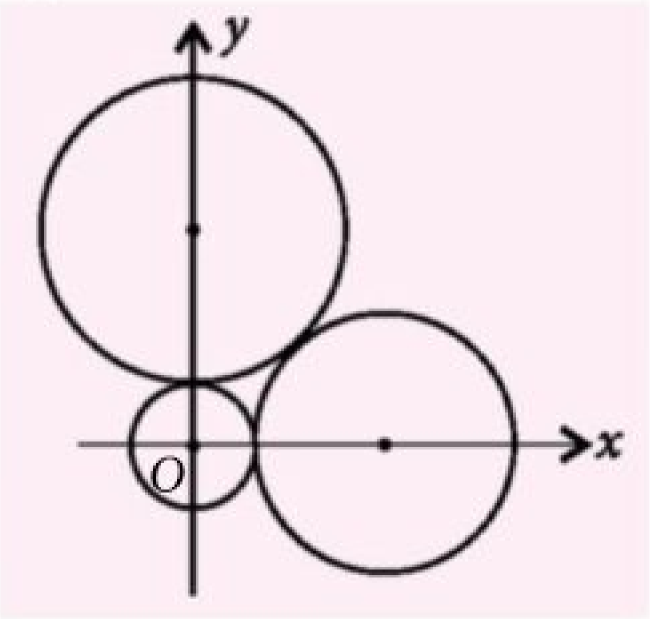
$g = \frac{1}{2}\left(\frac{d}{cx} - cx\right),\ f(1) = 1,\ g(2) = 3$

$\therefore\ f(x) = \frac{2}{x} - x,\ g(x) = \frac{2}{x} + x$

So, $f(2) + g(1) = 2$

8. (b) : $\int_1^2 f(x)\,g(x)\,dx = \int_1^2\left(\frac{4}{x^2} - x^2\right)dx = -\frac{1}{3}$

9. (6) : C_1 : centre (0, 0), radius 1
C_2 : centre (3, 0), radius 2
C_3 : centre (0, 4), radius 3
C_4 : centre (α, β), radius r
$\alpha^2 + \beta^2 = (r-1)^2$
$(\alpha-3)^2 + \beta^2 = (r-2)^2$,
$\alpha^2 + (\beta-4)^2 = (r-3)^2$
On solving, we get
$\alpha = 3, \beta = 4, r = 6$



10. (a) : P → 2; Q → 1; R → 4; S → 3

(P) Digits 1 to 9, $N = \binom{9}{4} = 126, S = 9$

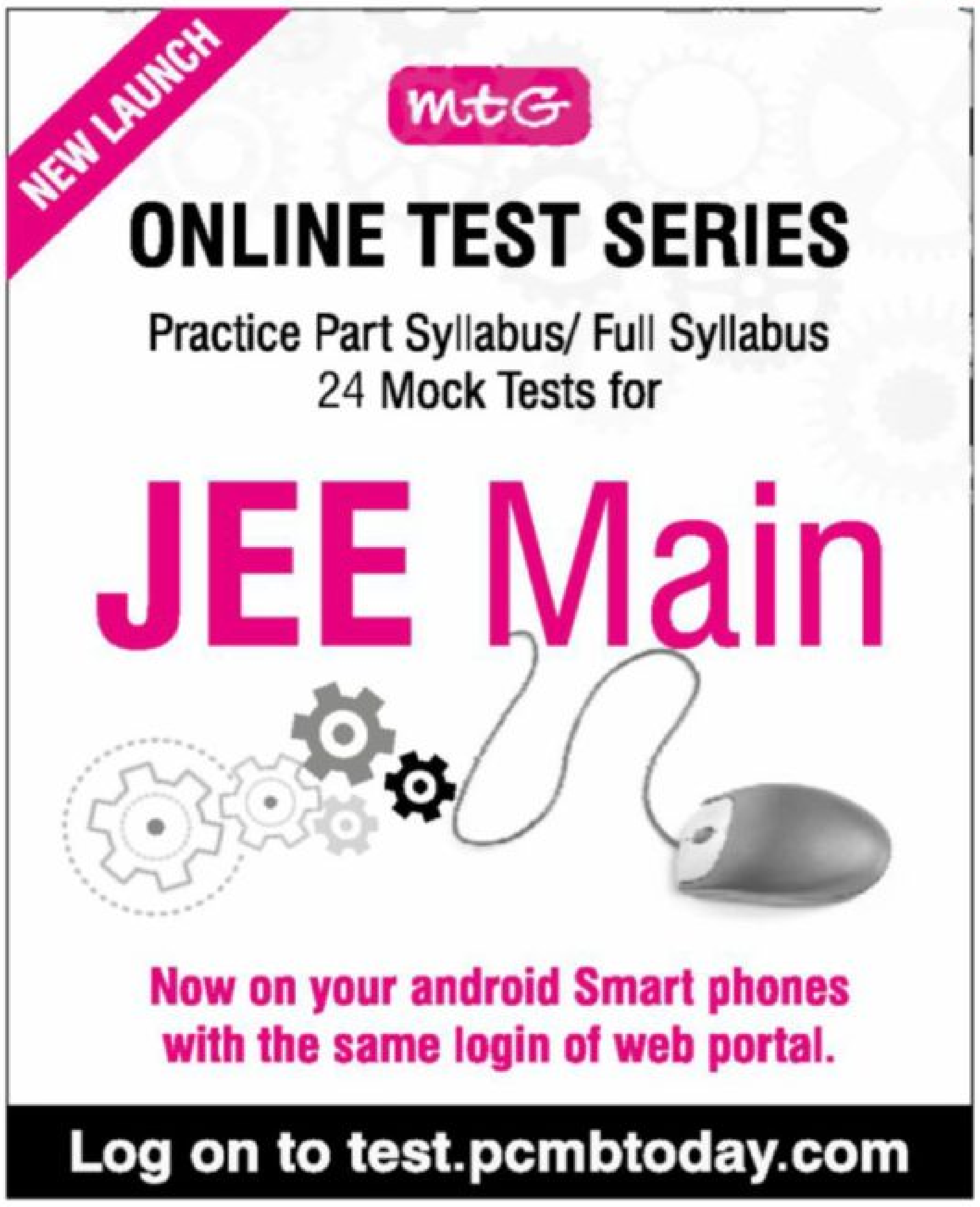
(Q) Digits 0 to 9, $N = \binom{10}{4} = 210, S = 3$

(R) Digits 1 to 9, $N = \binom{9+4-1}{4} = \binom{12}{4} = 495, S = 18$

(S) Digits 0 to 9,

$N = \binom{10+4-1}{4} - 1 = \binom{13}{4} - 1 = 714, S = 12$

Omitting 0000.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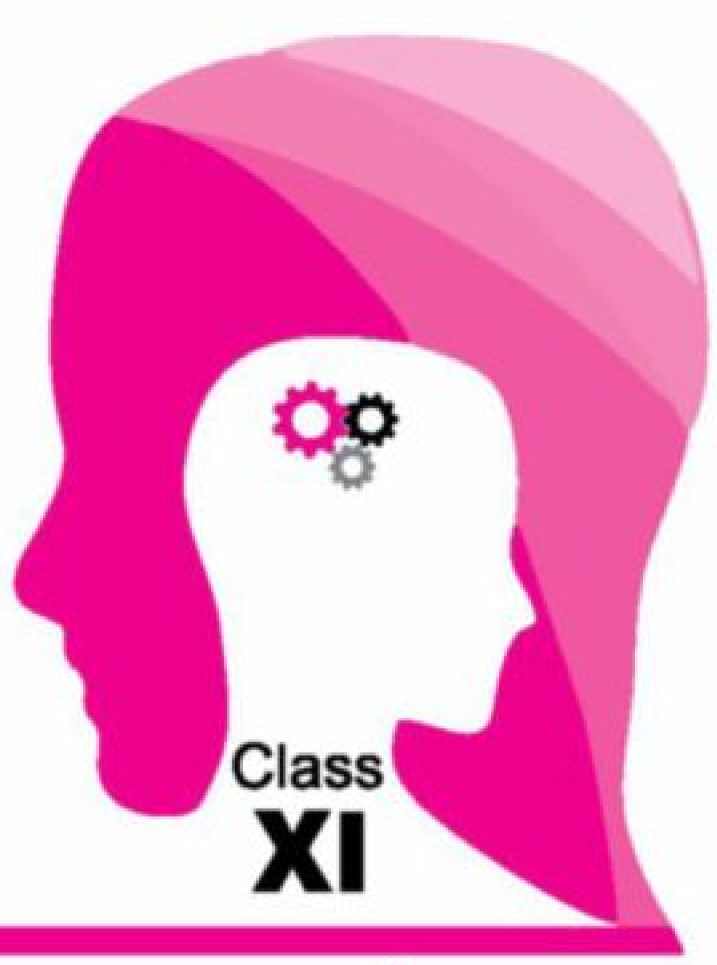
CONCEPT BOOSTER

Trigonometric Functions



*ALOK KUMAR

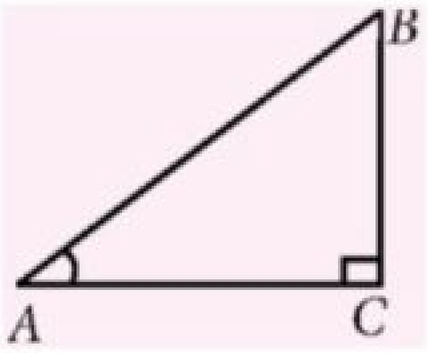
TRIGONOMETRIC RATIOS

- In a right angled triangle ABC, $\angle CAB = A$ and $\angle BCA = 90° = \pi/2$. AC is the base, BC is the altitude and AB is the hypotenuse. We refer to the base as the adjacent side and to the altitude as the opposite side. There are six trigonometric ratios, also called trigonometric functions or circular functions. With reference to angle A, the six ratios are:

  $\frac{BC}{AB} = \frac{\text{opposite side}}{\text{hypotenuse}}$ is called sine of A, and written as $\sin A$.

  $\frac{AC}{AB} = \frac{\text{adjacent side}}{\text{hypotenuse}}$ is called the cosine of A, and written as $\cos A$.

  $\frac{BC}{AC} = \frac{\text{opposite side}}{\text{adjacent side}}$ is called the tangent of A, and written as $\tan A$.



- Obviously, $\tan A = \frac{\sin A}{\cos A}$. The reciprocals of sine, cosine and tangent are called the cosecant, secant and cotangent of A respectively. We write these as $\operatorname{cosec} A$, $\sec A$ and $\cot A$ respectively.

Note :

- Since the hypotenuse is the greatest side in a right angle triangle so, $\sin A$ and $\cos A$ can never be greater than unity and $\operatorname{cosec} A$ and $\sec A$ can never be less than unity. Hence $|\sin A| \le 1$, $|\cos A| \le 1$, $|\operatorname{cosec} A| \ge 1$, $|\sec A| \ge 1$, while $\tan A$ and $\cot A$ may have any numerical value lying between $-\infty$ to $+\infty$.
- All the six trigonometric functions have got a very important property in common that is periodicity. Remember that the trigonometrical ratios are real numbers and remain same so long as the angle remains same.

Trigonometric identities

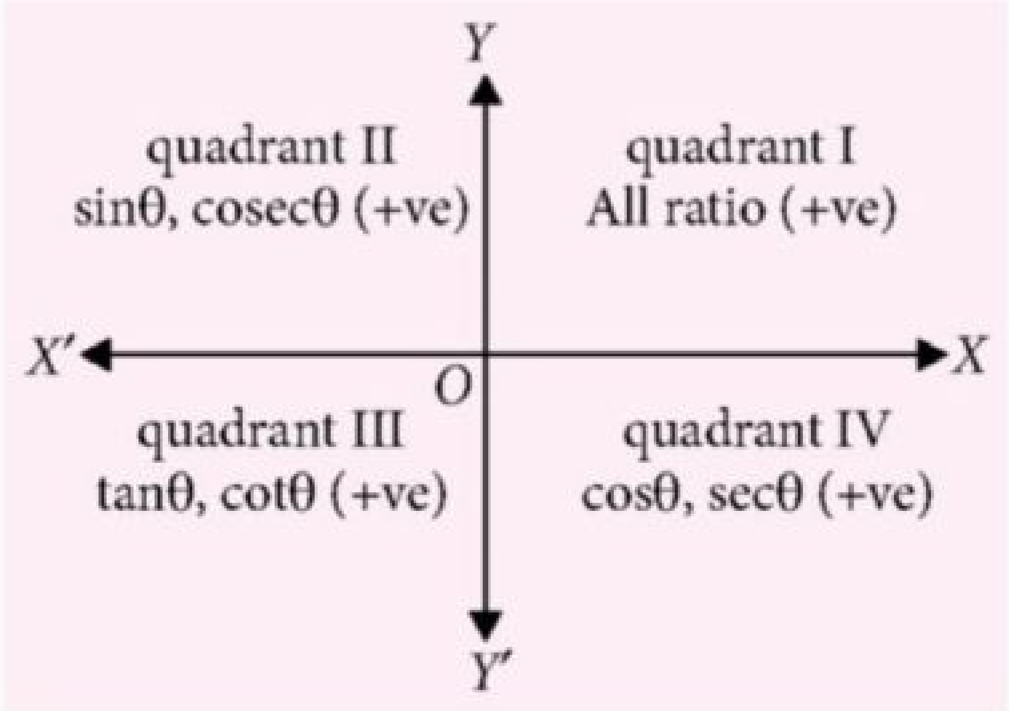
- $\sin^2 A + \cos^2 A = 1$
- $\sec^2 A - \tan^2 A = 1$
- $\operatorname{cosec}^2 A - \cot^2 A = 1$

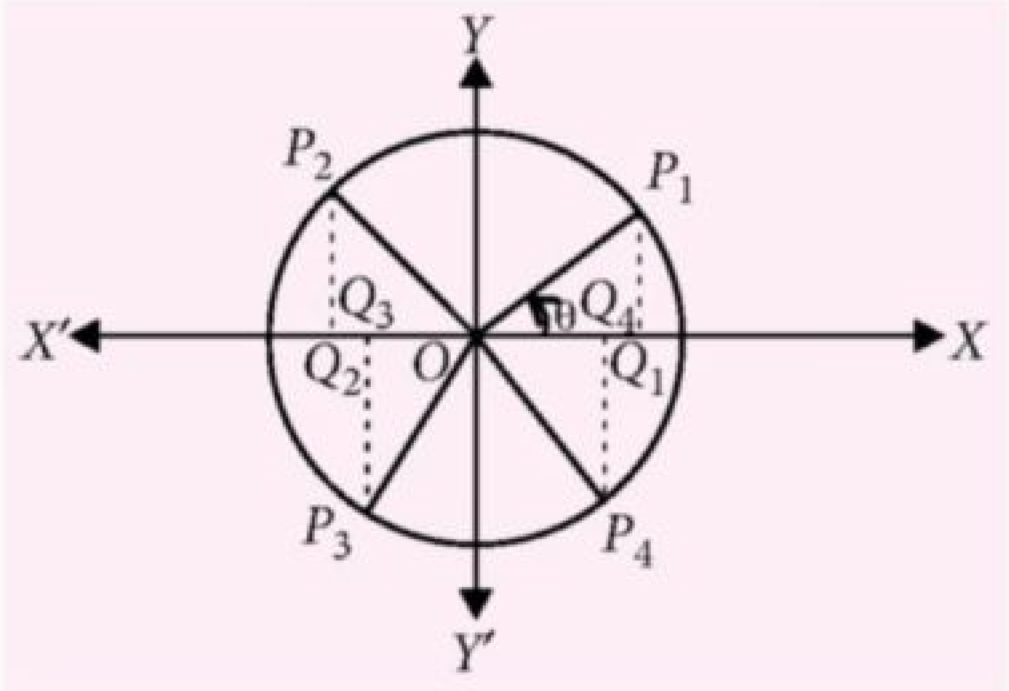
Fundamental inequality

- For $0 < A < \frac{\pi}{2}, 0 < \cos A < \frac{\sin A}{A} < \frac{1}{\cos A}$

TRIGONOMETRIC RATIOS OF ANY ANGLE

- Consider the system of rectangular co-ordinate axes dividing the plane into four quadrants. An angle θ lies in one and only one of these quadrants. Signs of the trigonometric ratios in the various quadrants are shown in the figure. Note that $\angle XOY = \pi/2$, $\angle XOX' = \pi$, $\angle XOY' = 3\pi/2$





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*Alok Kumar, a B.Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

P_iQ_i is positive if above the x-axis, negative if below the x-axis, OP_i is always taken positive. OQ_i is positive if along positive x-axis, negative if in opposite direction.

$$\sin\angle Q_iOP_i = \frac{P_iQ_i}{OP_i}, \quad \cos\angle Q_iOP_i = \frac{OQ_i}{OP_i}, \quad \tan\angle Q_iOP_i = \frac{P_iQ_i}{OQ_i} \quad (\text{where } i = 1, 2, 3, 4)$$

Thus depending on signs of OQ_i and P_iQ_i the various trigonometrical ratios will have different signs.

α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\operatorname{cosec}\alpha$
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$
$90° - \theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$
$90° + \theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$	$-\tan\theta$	$-\operatorname{cosec}\theta$	$\sec\theta$
$180° - \theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$	$-\cot\theta$	$-\sec\theta$	$\operatorname{cosec}\theta$
$180° + \theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$	$\cot\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$
$360° - \theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$
$360° + \theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$

- Angle θ and $90° - \theta$ are complementary angles, θ and $180° - \theta$ are supplementary angles
- $\sin(n\pi + (-1)^n\theta) = \sin\theta,\ n \in I$
- $\cos(2n\pi \pm \theta) = \cos\theta,\ n \in I$
- $\tan(n\pi + \theta) = \tan\theta,\ n \in I$
- sine of general angle of the form $n\pi + (-1)^n\theta$ will have same sign as that of sine of angle θ and so on. The same is true for the respective reciprocal functions also.

BASIC FORMULAE

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A.$
- $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$
  $= \cos^2 B - \sin^2 A.$
- $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
  $= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
- $\sin 3A = 3\sin A - 4\sin^3 A = 4\sin(60° - A)\sin A \sin(60° + A)$
- $\cos 3A = 4\cos^3 A - 3\cos A$
  $= 4\cos(60° - A)\cos A \cos(60° + A)$
- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
  $= \tan(60° - A)\tan A \tan(60° + A)$
- $\sin A + \sin B = 2\sin\dfrac{A + B}{2}\cos\dfrac{A - B}{2}$
- $\sin A - \sin B = 2\sin\dfrac{A - B}{2}\cos\dfrac{A + B}{2}$
- $\cos A + \cos B = 2\cos\dfrac{A - B}{2}\cos\dfrac{A + B}{2}$
- $\cos A - \cos B = 2\sin\dfrac{B - A}{2}\sin\dfrac{A + B}{2}$
- $\tan A + \tan B = \dfrac{\sin(A + B)}{\cos A \cdot \cos B}$
- $2\sin A \cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A \sin B = \sin(A + B) - \sin(A - B)$
- $2\cos A \cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

CONDITIONAL IDENTITIES

◆ For any angles A, B, C

- $\sin(A + B + C) = \sin A \cos B \cos C + \cos A \sin B \cos C$
  $+ \cos A \cos B \sin C - \sin A \sin B \sin C$

- $\cos(A+B+C)=\cos A\cos B\cos C-\cos A\sin B\sin C$
  $-\sin A\cos B\sin C-\sin A\sin B\cos C$
- $\tan(A+B+C)$
  $$=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan A\tan C}$$
- $\cot(A+B+C)$
  $$=\frac{\cot A\cot B\cot C-\cot A-\cot B-\cot C}{\cot A\cot B+\cot B\cot C+\cot A\cot C-1}$$
- If A, B, C are angles of a triangle (or $A+B+C=\pi$)
- $\sin A\cos B\cos C+\cos A\sin B\cos C+\cos A\cos B\sin C$
  $=\sin A\sin B\sin C$
- $\cos A\sin B\sin C+\sin A\cos B\sin C+\sin A\sin B\cos C$
  $=1+\cos A\cos B\cos C$
- $\tan A+\tan B+\tan C=\tan A\tan B\tan C$
- $\cot B\cot C+\cot C\cot A+\cot A\cot B=1$
- $\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$
- $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$
- $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$
- $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$
- $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$
- $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
- $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

TRIGONOMETRIC SERIES

- If we have a cosine series in its product form where the angles are in G.P. with common ratio 2, then multiply both numerator and denominator by 2 sin (least angle).
- $$\sum_{r=0}^{n-1}\sin(\alpha+r\beta)=\frac{\sin\left(\alpha+(n-1)\frac{\beta}{2}\right)\cdot\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$
- $$\sum_{r=0}^{n-1}\cos(\alpha+r\beta)=\frac{\cos\left(\alpha+(n-1)\frac{\beta}{2}\right)\cdot\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

TRIGONOMETRIC EQUATIONS

- An equation involving one or more trigonometrical ratios of unknown angle is called a trigonometric equation *e.g.*, $\cos^2 x-4\sin x=1$
- It is to be noted that a trigonometrical identity is satisfied for every value of the unknown angle where as trigonometric equation is satisfied only for some values (finite or infinite) of unknown angle.
  For example, $\sec^2 x-\tan^2 x=1$ is a trigonometrical identity as it is satisfied for every value of $x\in R$.

SOLUTION OF A TRIGONOMETRIC EQUATION

- A value of the unknown angle which satisfies the given equation is called a solution of the equation
  e.g. $\sin\theta=\frac{1}{2}\Rightarrow\theta=\pi/6$.

General Solution

- Since trigonometrical functions are periodic functions, solutions of trigonometric equations can be generalized with the help of the periodicity of the trigonometrical functions. The solution consisting of all possible solutions of a trigonometric equation is called its general solution.
- $\sin\theta=0\Rightarrow\theta=n\pi,\ n\in I$
- $\cos\theta=0\Rightarrow\theta=(2n+1)\frac{\pi}{2},\ n\in I$
- $\tan\theta=0\Rightarrow\theta=n\pi,\ n\in I$
- $\sin\theta=\sin\alpha\Rightarrow\theta=n\pi+(-1)^n\alpha,\ n\in I$
  where $\alpha\in[-\pi/2,\pi/2]$
- $\cos\theta=\cos\alpha\Rightarrow\theta=2n\pi\pm\alpha,\ n\in I$
  where $\alpha\in[0,\pi]$
- $\tan\theta=\tan\alpha\Rightarrow\theta=n\pi+\alpha, n\in I$
  where $\alpha\in(-\pi/2,\pi/2)$
- $\sin^2\theta=\sin^2\alpha,\ \cos^2\theta=\cos^2\alpha,\ \tan^2\theta=\tan^2\alpha$
  $\Rightarrow\theta=n\pi\pm\alpha,\ n\in I$
- $\sin\theta=1\Rightarrow\theta=(4n+1)\frac{\pi}{2},\ n\in I$
- $\cos\theta=1\Rightarrow\theta=2n\pi,\ n\in I$
- $\cos\theta=-1\Rightarrow\theta=(2n+1)\pi,\ n\in I$
- The general solution should be given unless the solution is required in a specified interval.
- α is taken as the principal value of the angle. Numerically least angle is called the principal value.

PROBLEMS

Single Correct Answer Type

1. $\sqrt{3}\,\mathrm{cosec}\,20^\circ - \sec 20^\circ =$
(a) 1 (b) 2 (c) 3 (d) 4

2. $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A =$
(a) $\cot A$ (b) $\tan 6A$ (c) $\cot 4A$ (d) $\cot 3A$

3. The value of $\sin 12^\circ \cdot \sin 48^\circ \cdot \sin 54^\circ =$
(a) 1/8 (b) 1/6 (c) 1/4 (d) 1/2

4. The smallest positive value of x (in degrees) for which $\tan(x + 100^\circ) = \tan(x + 50^\circ)\tan x \tan(x - 50^\circ)$ is
(a) 30° (b) 45° (c) 60° (d) 90°

5. Let $0 < A, B < \pi/2$ satisfying the equalities $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$. Then $A + 2B =$
(a) $\pi/4$ (b) $\pi/3$
(c) $\pi/2$ (d) none of these

6. If $a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$ and $a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$, then $(x + y)^{2/3} + (x - y)^{2/3} =$
(a) $2a^{2/3}$ (b) $a^{2/3}$ (c) $3a^{2/3}$ (d) $2a^{1/3}$

7. If $\cos^2\theta = \frac{1}{3}(a^2 - 1)$ and $\tan^2\frac{\theta}{2} = \tan^{2/3}\alpha$, then $\cos^{2/3}\alpha + \sin^{2/3}\alpha =$
(a) $2a^{2/3}$ (b) $\left(\frac{2}{a}\right)^{2/3}$ (c) $\left(\frac{2}{a}\right)^{1/3}$ (d) $2a^{1/3}$

8. The number of solutions of $\sin^3 x\cos x + \sin^2 x\cos^2 x + \sin x\cos^3 x = 1$ in $[0, 2\pi]$ is
(a) 4 (b) 2 (c) 1 (d) 0

9. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

10. If $\tan x = n \cdot \tan y$, $n \in R^+$, then maximum value of $\sec^2(x - y)$ is equal to
(a) $\frac{(n+1)^2}{2n}$ (b) $\frac{(n+1)^2}{n}$
(c) $\frac{(n+1)^2}{2}$ (d) $\frac{(n+1)^2}{4n}$

11. The minimum value of $\cos(\cos x)$ is
(a) 0 (b) $-\cos 1$ (c) $\cos 1$ (d) -1

12. If $S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + ... + \cos^2\frac{(n-1)\pi}{n}$, then S equals
(a) $\frac{n}{2}(n+1)$ (b) $\frac{1}{2}(n-1)$
(c) $\frac{1}{2}(n-2)$ (d) $\frac{n}{2}$

13. If $\tan 2\theta \cdot \tan\theta = 1$, then θ is equal to
(a) $n\pi + \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{6}$
(c) $2n\pi \pm \frac{\pi}{6}$ (d) none of these

14. If α is the root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\pi/2 < \alpha < \pi$, then $\sin 2\alpha$ is equal to
(a) $\frac{24}{25}$ (b) $\frac{-24}{25}$ (c) $\frac{13}{18}$ (d) $\frac{-13}{18}$

15. The equation $k\sin x + \cos 2x = 2k - 7$ possesses a solution if
(a) $k > 6$ (b) $2 \le k \le 6$
(c) $k > 2$ (d) none of these

16. The equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ is solvable if
(a) $-\sqrt{3} \le a \le \sqrt{3}$ (b) $-\sqrt{2} \le a \le \sqrt{2}$
(c) $-1 \le a \le 1$ (d) none of these

17. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is
(a) ϕ (b) $\pi/4$
(c) $n\pi + \frac{\pi}{4}; n = 1, 2, 3 .....$
(d) $2n\pi + \frac{\pi}{4}; n = 1, 2, 3 .....$

18. The value of the expression $\frac{1 - 4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$ is
(a) 1/2 (b) 1
(c) 2 (d) none of these

19. Number of solutions of $5\cos^2\theta - 3\sin^2\theta + 6\sin\theta\cos\theta = 7$ in the interval $[0, 2\pi]$ is
(a) 2 (b) 4
(c) 0 (d) none of these

20. If $\sqrt{3}\sin\pi x + \cos\pi x = x^2 - \frac{2}{3}x + \frac{19}{9}$, then x is equal to
(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) none of these

21. General solution for θ if
$\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right) = 2$, is
(a) $2n\pi + \frac{7\pi}{6}$ (b) $2n\pi + \frac{\pi}{6}$
(c) $2n\pi - \frac{7\pi}{6}$ (d) none of these

22. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

23. The general solution of the equation $(\sqrt{3}-1)\sin\theta+(\sqrt{3}+1)\cos\theta=2$ is

(a) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{12}$ (b) $2n\pi\pm\frac{\pi}{4}-\frac{\pi}{12}$

(c) $n\pi+(-1)^n\frac{\pi}{4}+\frac{\pi}{12}$ (d) $2n\pi\pm\frac{\pi}{4}+\frac{\pi}{12}$

24. One solution of the equation
$4\cos^2\theta\sin\theta - 2\sin^2\theta = 3\sin\theta$ is

(a) $x=n\pi+(-1)^n\left(\frac{-3\pi}{10}\right)$

(b) $x=n\pi+(-1)^n\left(\frac{3\pi}{10}\right)$

(c) $x=2n\pi\pm\frac{\pi}{6}$

(d) none of these

25. The solutions of the following equations are
$x\cos^3 y + 3x\cos y\sin^2 y = 14$
$x\sin^3 y + 3x\cos^2 y\sin y = 13$

(a) $y=\tan^{-1}\frac{1}{2}, x=5\sqrt{5}$ where $2n\pi<y<2n\pi+\frac{\pi}{2}$

(b) $y=\tan^{-1}\frac{1}{2}, x=-5\sqrt{5}$ where $2n\pi+\pi<y<2n\pi+\frac{3\pi}{2}$

(c) both (a) and (b) (d) none of these

Multiple Correct Answer Type

26. The solution of $\sin x+\sqrt{3}\cos x=\sqrt{2}$ is

(a) $2n\pi+\frac{5\pi}{12}$ (b) $2n\pi-\frac{\pi}{12}$

(c) $2n\pi\pm\frac{\pi}{4}$ (d) none of these

27. Which of the following quantities are rational?

(a) $\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$ (b) $\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$

(c) $\sin^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)$

(d) $\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$

28. If $x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta = 2a$ and $2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)=1$, then

(a) $y^2 = 4a(a - x)$

(b) $\cos\alpha + \cos\beta = \cos\alpha\cos\beta$

(c) $\cos\alpha\cos\beta=\frac{4a^2+y^2}{x^2+y^2}$

(d) $\cos\alpha+\cos\beta=\frac{4ax}{x^2+y^2}$

29. Let $f_n(\theta)=\tan\left(\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)$ ...$(1 + \sec 2^n\theta)$, then

(a) $f_2\left(\frac{\pi}{16}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_5\left(\frac{\pi}{128}\right)=1$

30. Let ABC be a triangle inscribed in a circle of radius r and $AB = AC$ and h is the altitude from A to BC, then (P = perimeter of ΔABC, Δ = area)

(a) $P=2\left(\sqrt{2hr-h^2}\right)$ (b) $\Delta=h\sqrt{2hr-h^2}$

(c) $\lim\limits_{h\to 0}\frac{\Delta}{P^3}=\frac{1}{128r}$ (d) $\lim\limits_{h\to 0}\frac{\Delta}{P^3}=\frac{1}{64r}$

31. Three straight lines are drawn through a point P lying in the interior of the triangle ABC and parallel to its sides. The area of the three resulting triangles with P as the vertex are Δ_1, Δ_2 and Δ_3, then area of triangle ABC is

(a) $\left(\sqrt{\Delta_1}+\sqrt{\Delta_2}+\sqrt{\Delta_3}\right)^2$

(b) $(\Delta_1 + \Delta_2 + \Delta_3)$

(c) $\left(\sqrt{\Delta_1\Delta_2+\Delta_2\Delta_3+\Delta_3\Delta_1}\right)$

(d) $\Delta_1+\Delta_2+\Delta_3+2\left(\sqrt{\Delta_1\Delta_2}+\sqrt{\Delta_2\Delta_3}+\sqrt{\Delta_3\Delta_1}\right)$

32. A solution (x, y) of the system of equations $x-y=\frac{1}{3}$ and $\cos^2(\pi x)-\sin^2(\pi y)=\frac{1}{2}$ is

(a) $\left(\frac{7}{6},\frac{5}{6}\right)$ (b) $\left(\frac{2}{3},\frac{1}{3}\right)$

(c) $\left(\frac{-5}{6},\frac{-7}{6}\right)$ (d) $\left(\frac{13}{6},\frac{11}{6}\right)$

33. For $0\le x\le 2\pi$, $2^{\operatorname{cosec}^2 x}\sqrt{\frac{y^2}{2}-y+1}\le\sqrt{2}$ is

(a) satisfied by exactly one value of y
(b) satisfied by exactly two values of x
(c) satisfied by x for which $\cos x = 0$
(d) satisfied by x for which $\sin x = 0$

34. If $\dfrac{\tan 3A}{\tan A}=k(k\neq 1)$ then

(a) $\dfrac{\cos A}{\cos 3A}=\dfrac{k-1}{2}$ (b) $\dfrac{\sin 3A}{\sin A}=\dfrac{2k}{k-1}$

(c) $k<\dfrac{1}{3}$ (d) $k>3$

Assertion & Reason Type

Direction : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement–2 is true, Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

35. Statement-1 : If $\theta\in\left(\dfrac{\pi}{3},\dfrac{\pi}{2}\right)$, then $\tan\theta^{\cot\theta}>\cot\theta^{\tan\theta}$

Statement-2 : The function $x^{1/x}$ decreases for all $x>3$.

36. Statement-1 : The inequality $\log_{\sin x}2^{\tan x}>0$ has no real roots in the interval $\left(0,\dfrac{\pi}{2}\right)$.

Statement-2 : The domain of the function $f(x)=\log_{\sin x}2^{\tan x}$ is $\bigcup\limits_{n\in I}\left(2n\pi, 2n\pi+\dfrac{\pi}{2}\right)$.

37. Statement-1 : The maximum value of $\sin\sqrt{2}x+\sin ax$ cannot be 2 (a is positive rational number).

Statement-2 : $\dfrac{\sqrt{2}}{a}$ is irrational.

Matrix-Match Type

38. Match the following.

	Column-I	Column-II	
A.	If $\tan\theta$ is the G.M. between $\sin\theta$ and $\cos\theta$, then $2-4\sin^2\theta+3\sin^4\theta-\sin^6\theta=$	p.	1
B.	$\sqrt{3}\cot 20^\circ-4\cos 20^\circ=$	q.	0
C.	$\cot 16^\circ\cot 44^\circ+\cot 44^\circ\cot 76^\circ-\cot 76^\circ\cot 16^\circ=$	r.	3
D.	$\sum\limits_{r=1}^{9}\sin^2\left(\dfrac{r\pi}{18}\right)=$	s.	5

39. Match the following.

	Column-I	Column-II	
A.	If $\sin\theta=3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is	p.	0
B.	If $p\sin\theta+q\cos\theta=a$ and $p\cos\theta-q\sin\theta=b$, then $\dfrac{p+a}{q+b}+\dfrac{q-b}{p-a}+1$ is equal to	q.	1
C.	The value of the expression $\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{10\pi}{7}-\sin\dfrac{\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{5\pi}{14}$ is	r.	$\sec\theta$
D.	If $\sec\theta+\tan\theta=1$, then one root of the equation $(a-2b+c)x^2+(b-2c+a)x+(c-2a+b)=0$ is	s.	$-1/4$
		t.	$-1/2$

Numerical Value Type

40. If $\dfrac{\sin\theta}{\cos 3\theta}+\dfrac{\sin 3\theta}{\cos 9\theta}+\dfrac{\sin 9\theta}{\cos 27\theta}=\dfrac{1}{A}[\tan B\theta-\tan C\theta]$, then $(27A-B-27C)=$

41. If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, then the value of $\cos^6\theta-4\cos^4\theta+8\cos^2\theta$ must be

42. If $\alpha=\dfrac{\pi}{14}$, then the value of $(\tan\alpha\tan 2\alpha+\tan 2\alpha\tan 4\alpha+\tan 4\alpha\tan\alpha)$ is

SOLUTIONS

1. (d) : Given, $\dfrac{\sqrt{3}}{\sin 20^\circ}-\dfrac{1}{\cos 20^\circ}=\dfrac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}$

$$=\frac{4\cdot\left(\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ\right)}{2\sin 20^\circ\cos 20^\circ}$$

$$=4\cdot\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin 40^\circ}=4\frac{\sin 40^\circ}{\sin 40^\circ}=4$$

2. (a) : $\tan A+2\tan 2A+4\tan 4A+8\cot 8A$

$=\tan A+2\tan 2A+4\tan 4A+8\dfrac{(1-\tan^2 4A)}{2\tan 4A}$

$=\tan A+2\tan 2A+4\cot 4A$

$$=\tan A+2\tan 2A+4\frac{(1-\tan^2 2A)}{2\tan 2A}$$

$$=\tan A+2\cot 2A=\tan A+\frac{2(1-\tan^2 A)}{2\tan A}=\cot A$$

3. (a) : $\sin 12°\cdot\sin 48°\cdot\sin 54°$

$$=\frac{1}{2}[\cos 36^\circ - \cos 60^\circ]\sin 54^\circ$$

$$=\frac{1}{2}\left[\cos 36^\circ \sin 54^\circ - \frac{1}{2}\sin 54^\circ\right]$$

$$=\frac{1}{4}[2\cos 36°\sin 54° - \sin 54°]$$

$$=\frac{1}{4}[\sin 90° + \sin 18° - \sin 54°]$$

$$=\frac{1}{4}[1-(\sin 54° - \sin 18°)]=\frac{1}{4}[1-2\sin 18^\circ \cos 36^\circ]$$

$$=\frac{1}{4}\left[1-\frac{2\sin 18^\circ}{\cos 18^\circ}\cos 18^\circ \cos 36^\circ\right]=\frac{1}{4}\left[1-\frac{\sin 36^\circ \cos 36^\circ}{\cos 18^\circ}\right]$$

$$=\frac{1}{4}\left[1-\frac{2\sin 36^\circ \cos 36^\circ}{2\cos 18^\circ}\right]=\frac{1}{4}\left[1-\frac{\sin 72^\circ}{2\sin 72^\circ}\right]=\frac{1}{8}$$

4. (a) : The given relation may be written as

$$\frac{\tan(x+100^\circ)}{\tan(x-50^\circ)}=\tan(x+50^\circ)\tan x$$

$$\Rightarrow \frac{\sin(x+100°)\cos(x-50°)}{\sin(x-50°)\cos(x+100°)}=\frac{\sin(x+50°)\sin x}{\cos(x+50°)\cos x}$$

$$\Rightarrow \frac{\sin(2x+50°)+\sin(150°)}{\sin(2x+50°)-\sin(150°)}=\frac{\cos(50°)-\cos(2x+50°)}{\cos(50°)+\cos(2x+50°)}$$

$$\Rightarrow \frac{\sin(2x+50°)}{\sin 150°}=\frac{-\cos 50°}{\cos(2x+50°)}$$

[Applying componendo and dividendo]

$\Rightarrow$ $\cos 50° + 2\sin(2x + 50°)\cos(2x + 50°) = 0$

$\Rightarrow$ $\cos 50° + \sin(4x + 100°) = 0$

$\Rightarrow$ $\cos 50° + \cos(4x + 10°) = 0$

$\Rightarrow$ $\cos(2x + 30°)\cos(2x - 20°) = 0 \Rightarrow x = 30°, 55°$

$\Rightarrow$ The smallest value of $x = 30°$

5. (c)

6. (a) : $a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$

$a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$

$x + y = a[\sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta + \cos\theta)]$

$= a(\sin\theta + \cos\theta)^3$

$$\Rightarrow \left(\frac{x+y}{a}\right)^{1/3}=\sin\theta+\cos\theta \qquad \text{...(i)}$$

$x - y = a[\cos^3\theta - \sin^3\theta + 3\cos\theta\sin^2\theta - 3\cos^2\theta\sin\theta]$

$= a[\cos\theta - \sin\theta]^3$

$$\Rightarrow \left(\frac{x-y}{a}\right)^{1/3}=\cos\theta-\sin\theta \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$(\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2 = \frac{(x+y)^{2/3}+(x-y)^{2/3}}{a^{2/3}}$$

$$\Rightarrow 2(\sin^2\theta+\cos^2\theta)=\frac{(x+y)^{2/3}+(x-y)^{2/3}}{a^{2/3}}$$

$\Rightarrow$ $(x + y)^{2/3} + (x - y)^{2/3} = 2a^{2/3}$

7. (b) : Given, $\cos^2\theta = \dfrac{a^2-1}{3}$, $\tan^2(\theta/2) = \tan^{2/3}\alpha$

$$\text{Now, } \tan^3\frac{\theta}{2}=\tan\alpha \Rightarrow \frac{\sin^3\frac{\theta}{2}}{\cos^3\frac{\theta}{2}}=\frac{\sin\alpha}{\cos\alpha}$$

$$\text{Let } \frac{\sin^3\frac{\theta}{2}}{\sin\alpha}=\frac{\cos^3\frac{\theta}{2}}{\cos\alpha}=k$$

$$\Rightarrow \sin^3\frac{\theta}{2}=k\sin\alpha \quad \text{... (i) and } \cos^3\frac{\theta}{2}=k\cos\alpha \quad \text{... (ii)}$$

From (i) and (ii), $k^{2/3}\sin^{2/3}\alpha + k^{2/3}\cos^{2/3}\alpha = 1$

$$\Rightarrow \sin^{2/3}\alpha + \cos^{2/3}\alpha = \frac{1}{k^{2/3}} \qquad \text{...(iii)}$$

Squaring (i) and (ii) and then adding, we get

$$k^2(\sin^2\alpha+\cos^2\alpha)=\sin^6\frac{\theta}{2}+\cos^6\frac{\theta}{2}$$

$$=\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right)^3-3\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right)$$

$$\Rightarrow k^2=1-\frac{3}{4}\sin^2\theta=1-\frac{3}{4}+\frac{3}{4}\cos^2\theta$$

$$\Rightarrow k^2=\frac{a^2}{4} \Rightarrow k=\frac{a}{2}$$

$$\therefore \text{ From (iii), we have } \sin^{2/3}\alpha+\cos^{2/3}\alpha=\left(\frac{2}{a}\right)^{2/3}$$

8. (d) : $\sin x\cos x[\sin^2x + \sin x\cos x + \cos^2x] = 1$

$\Rightarrow$ $\sin x\cos x + (\sin x\cos x)^2 = 1$

$\Rightarrow$ $\sin^2 2x + 2\sin 2x - 4 = 0$

$$\Rightarrow \sin 2x=\frac{-2\pm\sqrt{4+16}}{2}=-1\pm\sqrt{5}, \text{ which is not}$$

possible.

9. (a) : Let $f(x) = x^3 + 2x^2 + 5x + 2\cos x$

$\Rightarrow$ $f'(x) = 3x^2 + 4x + 5 - 2\sin x$

$$=3\left(x+\frac{2}{3}\right)^2+\frac{11}{3}-2\sin x$$

$$\text{Now, } \frac{11}{3}-2\sin x>0\ \forall x(\text{as } -1\le\sin x\le 1)$$

$\Rightarrow\ f'(x) > 0\ \forall\ x\ \Rightarrow f(x)$ is an increasing function. Now $f(0) = 2\ \Rightarrow f(x) = 0$ has no solution in $[0, 2\pi]$.

10. (d) : Given, $\tan x = n\tan y$

Now, $\cos(x - y) = \cos x \cos y + \sin x \sin y$

$\Rightarrow\ \cos(x - y) = \cos x \cos y(1 + \tan x \tan y)$

$= \cos x \cos y\ (1 + n\tan^2 y)$

$$\Rightarrow\ \sec^2(x-y) = \frac{\sec^2 x \sec^2 y}{(1+n\tan^2 y)^2}$$

$$= \frac{(1+\tan^2 x)(1+\tan^2 y)}{(1+n\tan^2 y)^2} = \frac{(1+n^2\tan^2 y)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= 1 + \frac{(n-1)^2\tan^2 y}{(1+n\tan^2 y)^2}$$

Now, $\left(\dfrac{1+n\tan^2 y}{2}\right)^2 \geq n\tan^2 y$

$$\Rightarrow\ \frac{\tan^2 y}{(1+n\tan^2 y)^2} \leq \frac{1}{4n}$$

$$\therefore\ \sec^2(x-y) \leq 1 + \frac{(n-1)^2}{4n} = \frac{(n+1)^2}{4n}$$

11. (c) : $\cos x$ varies from -1 to 1 for all real x. Thus $\cos(\cos x)$ varies from $\cos 1$ to $\cos 0 \Rightarrow$ minimum value of $\cos(\cos x)$ is $\cos 1$.

12. (c) : $S = \cos^2\dfrac{\pi}{n} + \cos^2\dfrac{2\pi}{n} + \ldots + \cos^2(n-1)\dfrac{\pi}{n}$

$$= \frac{1}{2}\left[1+\cos\frac{2\pi}{n} + 1 + \cos\frac{4\pi}{n} + 1 + \cos\frac{6\pi}{n} + \ldots + 1 + \cos 2(n-1)\frac{\pi}{n}\right]$$

$$= \frac{1}{2}\left[n - 1 + \sum_{k=1}^{n-1}\cos\frac{2k\pi}{n}\right] = \frac{1}{2}[n-1-1] = \frac{1}{2}(n-2)$$

13. (b) : $\tan 2\theta \cdot \tan\theta = 1$

$$\Rightarrow\ \frac{2\tan^2\theta}{1-\tan^2\theta} = 1\ \Rightarrow\ \tan^2\theta = \frac{1}{3}$$

$$\Rightarrow\ \tan^2\theta = \tan^2\frac{\pi}{6}\ \Rightarrow\ \theta = n\pi \pm \frac{\pi}{6}$$

14. (b) : Since, α is the root of $25\cos^2\theta + 5\cos\theta - 12 = 0$.

$\therefore\ 25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$$\Rightarrow\ \cos\alpha = \frac{-5 \pm \sqrt{25+1200}}{50}\ \Rightarrow\ \cos\alpha = \frac{-5\pm 35}{50}$$

$$\Rightarrow\ \cos\alpha = \frac{-4}{5} \qquad [\because\ \pi/2 < \alpha < \pi]$$

$$\therefore\ \sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{-24}{25}$$

15. (b) : We have, $k\sin x + (1 - 2\sin^2 x) = 2k - 7$

$\Rightarrow\ 2\sin^2 x - k\sin x + 2(k - 4) = 0$

$$\Rightarrow\ \sin x = \frac{k \pm \sqrt{k^2 - 16k + 64}}{4}$$

$$= \frac{k \pm |k-8|}{4} = \frac{1}{2}(k-4), 2$$

But $\sin x \neq 2$, therefore, $\sin x = \dfrac{1}{2}(k-4)$

Now, $-1 \leq \sin x \leq 1\ \Rightarrow\ -1 \leq \dfrac{k-4}{2} \leq 1\ \Rightarrow\ 2 \leq k \leq 6$

16. (b) : We have $\sin^4 x - 2\cos^2 x + a^2 = 0$

$\Rightarrow\ y^2 - 2(1 - y) + a^2 = 0$, where $\sin^2 x = y$

$\Rightarrow\ y^2 + 2y + a^2 - 2 = 0\ \Rightarrow\ y = -1 \pm \sqrt{3-a^2}$

For y to be real, $a^2 \leq 3$ ...(i)

But $\sin^2 x = y$, therefore $0 \leq y \leq 1$

$\Rightarrow\ 0 \leq -1 + \sqrt{3-a^2} \leq 1\ \Rightarrow\ 1 \leq \sqrt{3-a^2} \leq 2$

$\Rightarrow\ 1 \leq 3 - a^2 \leq 4\ \Rightarrow\ 2 - a^2 \geq 0 \Rightarrow a^2 \leq 2$ ...(ii)

From (i) and (ii), $a^2 \leq 2\ \Rightarrow\ -\sqrt{2} \leq a \leq \sqrt{2}$

17. (a) : $\tan(3x - 2x) = \tan x = 1$

$\therefore\ x = n\pi + (\pi/4)$ but this value does not satisfy the given equation as $\tan 2x = \tan\left(\dfrac{\pi}{2}\right) = \infty$ and it reduces to indeterminate form.

18. (b) : Given expression is

$$\frac{1-2[\cos 60^\circ - \cos 80^\circ]}{2\sin 10^\circ} = \frac{1-2\left[\frac{1}{2} - \cos 80^\circ\right]}{2\sin 10^\circ}$$

$$= \frac{2\cos 80^\circ}{2\sin 10^\circ} = \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} = \frac{\sin 10^\circ}{\sin 10^\circ} = 1$$

19. (c) : $5\cos^2\theta - 3\sin^2\theta + 6\sin\theta\cos\theta = 7$

$$\Rightarrow\ 5\left(\frac{1+\cos 2\theta}{2}\right) - 3\left(\frac{1-\cos 2\theta}{2}\right) + 3\sin 2\theta = 7$$

$\Rightarrow\ 4\cos 2\theta + 3\sin 2\theta = 6$

But $4\cos 2\theta + 3\sin 2\theta \leq \sqrt{4^2 + 3^2} = 5$

$\therefore$ Solution does not exist.

20. (b) : L.H.S. $= \sqrt{3}\sin\pi x + \cos\pi x = 2\sin\left(\pi x + \dfrac{\pi}{6}\right) \leq 2$

and equality holds for $x = 1/3$

and R.H.S. $= x^2 - \frac{2}{3}x + \frac{19}{9} = \left(x - \frac{1}{3}\right)^2 + 2 \geq 2$

Equality holds if $x = 1/3$.
Thus, L.H.S. = R.H.S. for $x = 1/3$ only.

21. (a) : $\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right) = 2$ ... (i)

$\because\ \sin\left(2\theta + \frac{\pi}{6}\right) \leq 1$ and $\cos\left(\theta + \frac{5\pi}{6}\right) \leq 1$

$\therefore$ (i) may holds true iff $\sin\left(2\theta + \frac{\pi}{6}\right)$ and $\cos\left(\theta + \frac{5\pi}{6}\right)$ both equal to 1 simultaneously. First common value of θ is $7\pi/6$ for which

$$\sin\left(2\theta + \frac{\pi}{6}\right) = \sin\left(\frac{14\pi}{6} + \frac{\pi}{6}\right) = \sin\frac{5\pi}{2} = \sin\frac{\pi}{2} = 1$$

and $\cos\left(\theta + \frac{5\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + \frac{5\pi}{6}\right) = \cos 2\pi = 1$

and since periodicity of $\sin\left(2\theta + \frac{\pi}{6}\right)$ is π

and periodicity of $\cos\left(\theta + \frac{5\pi}{6}\right)$ is 2π, therefore,

periodicity of $\sin\left(2\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{5\pi}{6}\right)$ is 2π.

Therefore, general solution is $\theta = 2n\pi + \frac{7\pi}{6}$.

22. (c) : The given equation can be written as
$\sin x + 1 = 2\cos^2 x \Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$
$\Rightarrow\ 2\sin^2 x + \sin x - 1 = 0$

$\therefore\ \sin x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \frac{1}{2}$ or -1

$\Rightarrow\ x = \frac{\pi}{6}, \frac{5\pi}{6}$ [$\because$ $x = 3\pi/2$ does not satisfy given equation]

Hence, the required number of solutions is 2.

23. (d) : Let $\sqrt{3} + 1 = r\cos\alpha, \sqrt{3} - 1 = r\sin\alpha$

$\therefore\ r^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 = 8 \Rightarrow r = 2\sqrt{2}$

and $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \tan(45° - 30°)$

$\Rightarrow\ \tan\alpha = \tan 15° \Rightarrow \alpha = 15° = \frac{\pi}{12}$

Using these in the given equation, we get
$r\cos(\theta - \alpha) = 2$

$\Rightarrow\ \cos\left(\theta - \frac{\pi}{12}\right) = \frac{2}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$

$\therefore\ \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}, n \in I$

24. (a) : The given equation can be written as
$\sin\theta[4(1 - \sin^2\theta) - 2\sin\theta - 3] = 0$
$\Rightarrow\ \sin\theta[1 - 2\sin\theta - 4\sin^2\theta] = 0$
$\Rightarrow\ \sin\theta[4\sin^2\theta + 2\sin\theta - 1] = 0$
$\Rightarrow$ Either $\sin\theta = 0$ which gives $\theta = n\pi$
or $4\sin^2\theta + 2\sin\theta - 1 = 0$ which gives

$$\sin\theta = \frac{-2 \pm \sqrt{4+16}}{2 \times 4} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$= \frac{-1+\sqrt{5}}{4}, \frac{-1-\sqrt{5}}{4}$$

Now, $\sin\theta = \frac{1}{4}(\sqrt{5} - 1) = \sin 18° = \sin\left(\frac{\pi}{10}\right)$

$\therefore\ \theta = n\pi + (-1)^n\left(\frac{\pi}{10}\right)$

Again, $\sin\theta = -\frac{1}{4}(\sqrt{5}+1) = -\cos 36°$

$$= -\cos(90° - 54°) = -\sin 54° = \sin(-54°) = \sin\left(\frac{-3\pi}{10}\right)$$

$\therefore\ \theta = n\pi + (-1)^n\left(-\frac{3\pi}{10}\right)$

25. (c) : Clearly, $x \neq 0$, so dividing the equations, we get

$$\frac{\cos^3 y + 3\cos y \sin^2 y}{\sin^3 y + 3\cos^2 y \sin y} = \frac{14}{13}$$

Applying componendo and dividendo, we get

$$\frac{(\cos y + \sin y)^3}{(\cos y - \sin y)^3} = \frac{14+13}{14-13} \Rightarrow \left(\frac{\cos y + \sin y}{\cos y - \sin y}\right)^3 = 27 = 3^3$$

$\Rightarrow\ \frac{\cos y + \sin y}{\cos y - \sin y} = \frac{3}{1}$

$\Rightarrow\ \frac{1 + \tan y}{1 - \tan y} = \frac{3}{1} \Rightarrow 4\tan y = 2 \Rightarrow \tan y = \frac{1}{2}$

$\Rightarrow\ \sin y = \frac{1}{\sqrt{5}}, \cos y = \frac{2}{\sqrt{5}}$ (when y is in 1st quadrant)

and $\sin y = -\frac{1}{\sqrt{5}}, \cos y = -\frac{2}{\sqrt{5}}$ (when y is in 3rd quadrant)

When y is in first quadrant, we have

$$x\left[\frac{8}{5\sqrt{5}} + 3\frac{2}{\sqrt{5}}\frac{1}{5}\right] = 14 \Rightarrow x = 5\sqrt{5}$$

When y is in 3^{rd} quadrant, we have

$$x\left[\frac{-8}{5\sqrt{5}}+3\left(\frac{-2}{\sqrt{5}}\right)\frac{1}{5}\right]=14 \Rightarrow x=-5\sqrt{5}$$

Hence, $y=\tan^{-1}\frac{1}{2}, x=5\sqrt{5},$ where $2n\pi < y < 2n\pi+\frac{\pi}{2}$

and $y=\tan^{-1}\frac{1}{2}, x=-5\sqrt{5},$

where $2n\pi + \pi < y < 2n\pi+\frac{3\pi}{2}$

26. (a, b) : Given, $\sqrt{3}\cos x+\sin x=\sqrt{2}$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\frac{\sqrt{2}}{2}$$

$$\Rightarrow \cos\left(x-\frac{\pi}{6}\right)=\cos\frac{\pi}{4} \Rightarrow x-\frac{\pi}{6}=2n\pi\pm\frac{\pi}{4}$$

$$\Rightarrow x=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{6},\ n\in I$$

$\Rightarrow x=2n\pi+\frac{5\pi}{12},\ 2n\pi-\frac{\pi}{12},$ where $n \in I$.

27. (a, b, c, d) : (a) $\sin\frac{11\pi}{12}\sin\frac{5\pi}{12}$

$$=\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)=\frac{1}{2}\sin\left(\frac{\pi}{6}\right)=\frac{1}{4}\in Q$$

(b) $\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)=-\operatorname{cosec}\left(\frac{\pi}{10}\right)\sec\left(\frac{\pi}{5}\right)=-4\in Q$

(c) $1-2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)=1-\frac{1}{4}=\frac{3}{4}\in Q$

(d) $\left(2\cos^2\frac{\pi}{9}\right)\left(2\cos^2\frac{2\pi}{9}\right)\left(2\cos^2\frac{4\pi}{9}\right)=\frac{1}{8}\in Q$

28. (a, b, d) : α and β satisfy $x\cos\theta + y\sin\theta = 2a$

$\Rightarrow (x^2 + y^2)\cos^2\theta - 4ax\cos\theta + (4a^2 - y^2) = 0$

$$\cos\alpha+\cos\beta=\frac{4ax}{x^2+y^2},\ \cos\alpha\cdot\cos\beta=\frac{4a^2-y^2}{x^2+y^2}$$

$$2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)=1\Rightarrow 4\sin^2\left(\frac{\alpha}{2}\right)\sin^2\left(\frac{\beta}{2}\right)=1$$

$\Rightarrow \cos\alpha + \cos\beta = \cos\alpha \cdot \cos\beta$

29. (a, b, c, d) : $f_n(\theta)=\tan\left(\frac{\theta}{2}\right)\prod_{r=0}^{n}(1+\sec 2^r\theta)$

$$=\tan\left(\frac{\theta}{2}\right)\prod_{r=0}^{n}\left\{\frac{1+\cos(2^r\theta)}{\cos(2^r\theta)}\right\}=\tan\left(\frac{\theta}{2}\right)\prod_{r=0}^{n}\frac{2\cos^2(2^{r-1}\theta)}{\cos(2^r\theta)}$$

$$=2^{n+1}\cdot\tan\left(\frac{\theta}{2}\right)\prod_{r=0}^{n}\frac{\cos^2(2^{r-1}\theta)}{\cos(2^r\theta)}$$

$$=2^{n+1}\cdot\tan\left(\frac{\theta}{2}\right)\cdot\frac{\cos^2(\theta/2)}{\cos(2^n\theta)}\prod_{r=0}^{n-1}\cos(2^r\theta)$$

$$=2^n\cdot\sin\theta\cdot\frac{\sin(2^n\theta)}{2^n\cdot\sin\theta\cdot\cos(2^n\theta)}=\tan(2^n\theta)$$

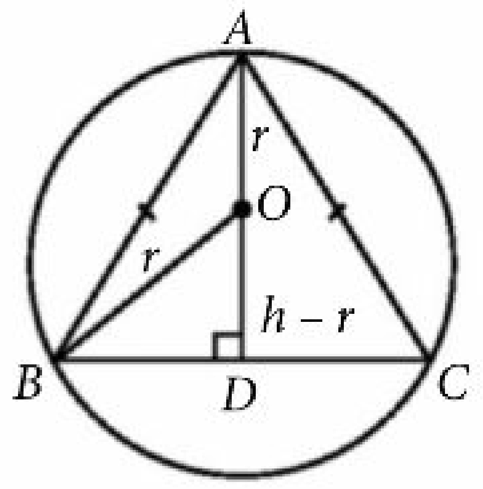
$$f_2\left(\frac{\pi}{16}\right)=\tan\left(\frac{\pi}{4}\right)=1;\quad f_3\left(\frac{\pi}{32}\right)=\tan\left(\frac{\pi}{4}\right)=1$$

$$f_4\left(\frac{\pi}{64}\right)=\tan\left(\frac{\pi}{4}\right)=1;\quad f_5\left(\frac{\pi}{128}\right)=\tan\left(\frac{\pi}{4}\right)=1$$

30. (b, c) : $BC=2BD=2\sqrt{r^2-(h-r)^2}=2\sqrt{2hr-h^2}$

$\Rightarrow AB=\sqrt{2hr}$ so that $P = 2AB + BC$

$$=2\left[\sqrt{2hr-h^2}+\sqrt{2hr}\right]$$

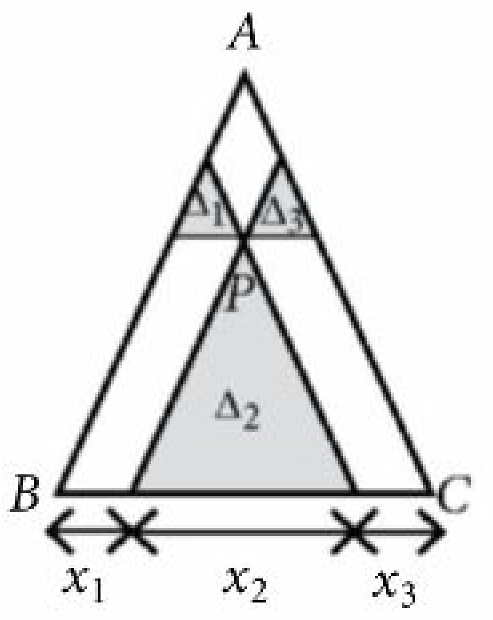


$$\Delta=BD\times AD=h\sqrt{2hr-h^2}$$

$$\therefore \frac{\Delta}{P^3}=\frac{\sqrt{2r-h}}{8\left(\sqrt{2r-h}+\sqrt{2r}\right)^3}$$

$$\Rightarrow \lim_{h\to 0}\frac{\Delta}{P^3}=\frac{\sqrt{2r}}{8(2\sqrt{2r})^3}=\frac{1}{128r}$$

31. (a, d) : We have, $\frac{\Delta_1}{\Delta}=\frac{x_1^2}{a^2}$



or $\sqrt{\frac{\Delta_1}{\Delta}}=\frac{x_1}{a}$

Similarly,

$\sqrt{\frac{\Delta_2}{\Delta}}=\frac{x_2}{a}$ and $\sqrt{\frac{\Delta_3}{\Delta}}=\frac{x_3}{a}$

32. (a, c, d) : We have, $x-y=\frac{1}{3}$ and

$\cos\{\pi(x+y)\}\cos\{\pi(x-y)\}=\frac{1}{2} \Rightarrow x+y=2n, n\in I$

33. (a, b, c) : $2^{\operatorname{cosec}^2 x}\sqrt{(y-1)^2+1}\le 2$

$\Rightarrow$ $\operatorname{cosec}^2 x = 1$ and $y = 1 \Rightarrow$ $x=\frac{\pi}{2},\frac{3\pi}{2}, y=1$

34. (a, b, c, d) : We have, $\frac{k+1}{k-1}=2\cos 2A$

Now, $\frac{\sin 3A}{\sin A}=1+2\cos 2A=\frac{2k}{k-1} \Rightarrow \frac{\cos A}{\cos 3A}=\frac{k-1}{2}$

Also, $k=\frac{3-\tan^2 A}{1-3\tan^2 A} \Rightarrow k<\frac{1}{3}, k>3$

35. (b) : Clearly, $f(x) = x^{1/x}$ decreases when $x > e$

Now, $\left(\frac{\pi}{3},\frac{\pi}{2}\right)\in(0, e) \Rightarrow \tan\theta>\cot\theta$

$\Rightarrow \quad (\tan\theta)^{\cot\theta} > (\cot\theta)^{\tan\theta}$

For $x > 3$, $f(x)$ decreases but it is not correct explanation for Statement 1.

36. (c) : $\log_{\sin x} 2^{\tan x} > 0 \Rightarrow 2^{\tan x} < 1$

i.e., $\tan x < 0$. But $\sin x > 0$. Hence, no solution.

37. (a) : The value of $\sin\sqrt{2}x + \sin ax$ can be equal to 2, if $\sin\sqrt{2}x$ and $\sin ax$ both are equal to one but are not equal to one for any common value of x.

38. $\mathbf{A \to p; B \to p; C \to r; D \to s}$

(A) $\tan^2\theta = \sin\theta\cos\theta \Rightarrow \sin\theta = \cos^3\theta$

$\therefore \quad (1 - \sin^2\theta) + (1 - 3\sin^2\theta) + 3\sin^4\theta - \sin^6\theta$

$= \cos^2\theta + (1 - \sin^2\theta)^3 = \cos^2\theta + \cos^6\theta$

$= \cos^2\theta + \sin^2\theta = 1$

(B) $\sin 40° = \sin(60° - 20°)$

$$\Rightarrow \quad 2\sin 20^\circ \cos 20^\circ = \frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ$$

$$\Rightarrow \quad 4\cos 20^\circ = \sqrt{3}\cot 20^\circ - 1$$

(C) $$\frac{3 + \cot 76^\circ \cot 16^\circ}{\cot 76^\circ + \cot 16^\circ} = \frac{3\sin 76^\circ \sin 16^\circ + \cos 76^\circ \cos 16^\circ}{\sin(76^\circ + 16^\circ)}$$

$$= \frac{2\sin 76^\circ \sin 16^\circ + \cos(76^\circ - 16^\circ)}{\sin(76^\circ + 16^\circ)}$$

$$= \frac{\cos 60^\circ - \cos 92^\circ + \cos 60^\circ}{\sin 92^\circ} = \frac{1 - \cos 92^\circ}{\sin 92^\circ} = \tan 46^\circ$$

$= \cot 44°$

(D) $$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{2\pi}{18}\right) + \ldots + \sin^2\left(\frac{\pi}{2}\right) = 5$$

39. $\mathbf{A \to p\,;\, B \to q\,;\, C \to s\,;\, D \to q, r}$

(A) Given, $\sin\theta = 3\sin(\theta + 2\alpha)$

$\Rightarrow \quad \sin(\theta + \alpha - \alpha) = 3\sin(\theta + \alpha + \alpha)$

$\Rightarrow \quad \sin(\theta + \alpha)\cos\alpha - \cos(\theta + \alpha)\sin\alpha$

$\qquad = 3\sin(\theta + \alpha)\cos\alpha + 3\cos(\theta + \alpha)\sin\alpha$

$\Rightarrow \quad -2\sin(\theta + \alpha)\cos\alpha = 4\cos(\theta + \alpha)\sin\alpha$

$$\Rightarrow \quad -\frac{\sin(\theta + \alpha)}{\cos(\theta + \alpha)} = \frac{2\sin\alpha}{\cos\alpha}$$

$\Rightarrow \quad -\tan(\theta + \alpha) = 2\tan\alpha \quad \Rightarrow \tan(\theta + \alpha) + 2\tan\alpha = 0$

(B) We have, $p\sin\theta + q\cos\theta = a$ ...(i)

And, $p\cos\theta - q\sin\theta = b$ ...(ii)

Squaring (i) and (ii), and then adding, we get

$(p\sin\theta + q\cos\theta)^2 + (p\cos\theta - q\sin\theta)^2 = a^2 + b^2$

$\Rightarrow \quad p^2(1) + q^2(1) - a^2 - b^2 = 0$

$\Rightarrow \quad (p^2 - a^2) + (q^2 - b^2) = 0$

$\Rightarrow \quad (p + a)(p - a) + (q - b)(q - b) = 0$

$$\Rightarrow \quad \frac{p + a}{q + b} + \frac{q - b}{p - a} = 0$$

(C) $$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$$

$$= \cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)$$

$$= -\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

Also, $\cos\frac{10\pi}{7} = \cos\frac{4\pi}{7}$

So, $$\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7} - \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$$

$$= 2\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right) = \frac{-1}{4}$$

(D) Clearly, $\sec\theta - \tan\theta = 1 \Rightarrow \sec\theta = 1$

Also, 1 satisfy the given equation.

So, one root of the given equation is 1 *i.e.*, $\sec\theta$.

40. (0) : $$\frac{\sin\theta}{\cos 3\theta} = \frac{2\sin\theta\cos\theta}{2\cos 3\theta\cos\theta} = \frac{\sin 2\theta}{2\cos 3\theta\cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{2\cos 3\theta\cos\theta}$$

$$\therefore \quad \frac{\sin\theta}{\cos 3\theta} = \frac{1}{2}[\tan 3\theta - \tan\theta] \qquad \text{...(i)}$$

$$\text{Similarly,} \quad \frac{\sin 3\theta}{\cos 9\theta} = \frac{1}{2}[\tan 9\theta - \tan 3\theta] \qquad \text{...(ii)}$$

$$\text{And} \quad \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}[\tan 27\theta - \tan 9\theta] \qquad \text{...(iii)}$$

Adding (i), (ii) and (iii), we get

$$\frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta} = \frac{1}{2}[\tan 27\theta - \tan\theta]$$

$\Rightarrow \quad A = 2,\ B = 27,\ C = 1 \quad \therefore \quad 27A - B - 27C = 0$

41. (4) : We have, $\sin\theta(1 + \sin^2\theta) = 1 - \sin^2\theta$

$\Rightarrow \quad \sin\theta(2 - \cos^2\theta) = \cos^2\theta$

Squaring both sides, we get

$\quad \sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow \quad (1 - \cos^2\theta)(4 - 4\cos^2\theta + \cos^4\theta) = \cos^4\theta$

$\Rightarrow \quad -\cos^6\theta + 5\cos^4\theta - 8\cos^2\theta + 4 = \cos^4\theta$

$\therefore \quad \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

42. (1) : $\alpha + 2\alpha + 4\alpha = 7\alpha = \frac{\pi}{2}$

$$\therefore \quad \tan\frac{\pi}{2} = \tan(\alpha + 2\alpha + 4\alpha)$$

$$= \frac{\tan\alpha + \tan 2\alpha + \tan 4\alpha - \tan\alpha\ \tan 2\alpha\ \tan 4\alpha}{1 - \tan\alpha\ \tan 2\alpha - \tan\alpha\ \tan 4\alpha - \tan 2\alpha\ \tan 4\alpha}$$

$\Rightarrow \quad \tan\alpha\ \tan 2\alpha + \tan 2\alpha\ \tan 4\alpha + \tan\alpha\ \tan 4\alpha = 1$

◆

CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 8

Introduction to Three Dimensional Geometry / Limits and Derivatives/Mathematical Reasoning

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All the questions are compulsory.

(ii) This question paper contains 36 questions.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION - A

Q.1 - Q.10 are multiple choice type questions. Select the correct option.

1. Let A, B, C be the feet of the perpendicular segments drawn from a point $P(3, 4, 5)$ on the xy, yz and zx-planes, respectively. What are the coordinates of A, B and C ?

(a) (3, 4, 0), (0, 4, 4), (3, 0, 5)
(b) (3, 0, 4), (4, 5, 0), (3, 5, 0)
(c) (3, 5, 0), (0, 5, 4), (0, 3, 4)
(d) (3, 4, 0), (0, 4, 5), (3, 0, 5)

2. $\lim_{k\to\infty}\left(\frac{1^3+2^3+3^3+\ldots+k^3}{k^4}\right)$ is equal to

(a) 0 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

3. Which of the following sentences is a statement?

(a) 5 is less than 7. (b) Where are you going?
(c) Close the door. (d) How funny he is!

4. The conditional statement of "You will get a sweet dish after the dinner" is

(a) If you take the dinner, then you will get a sweet dish.
(b) If you take the dinner, you will get a sweet dish.
(c) You get a sweet dish if and only if you take the dinner.
(d) None of these

5. $\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\left(\frac{\pi}{4}-x\right)(\cos x+\sin x)}$ is equal to

(a) 0 (b) 1 (c) – 1 (d) 2

6. If a parallelepiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes, then find the length of the diagonal.

(a) 7 units (b) 5 units (c) 8 units (d) 3 units

7. For the statement "19 is a real number or a positive integer", "Or" is

(a) inclusive (b) exclusive
(c) both (a) and (b) (d) none of these

8. If $y = x^2 + \sin x + \frac{1}{x^2}$, then $\frac{dy}{dx}$ is equal to
(a) $2x - \cos x + 2$ (b) $x - 2x^2 + \cos x$
(c) $2x + \cos x - (2/x^3)$ (d) $(2/x^3) - \cos x$

9. The point which divides the line joining the points (1, 3, 4) and (4, 3, 1) internally in the ratio 2 : 1, is
(a) (2, – 3, 3) (b) (2, 3, 3)
(c) $\left(\frac{5}{2}, 3, \frac{5}{2}\right)$ (d) (3, 3, 2)

10. If $f(x) = x \sin x$, then $f'\left(\frac{\pi}{2}\right)$ is equal to
(a) 0 (b) 1 (c) –1 (d) 2

(Q.11 - Q.15) Fill in the blanks.

11. The three planes determine a rectangular parallelepiped which has _______ of rectangular faces.

12. The positive integer n so that $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$ is ________.

OR

Let $f(x) = x - [x]; x \in R$, then $f'\left(\frac{1}{2}\right)$ is _______.

13. The perpendicular distance of the point (6, 5, 8) from z-axis is ________.

14. The sentence "There are 35 days in a month" is ________.

OR

The quantifier in the statement "For all real numbers x and y, $xy = yx$" is _________.

15. $\lim_{x \to 0} \left(x \sin \frac{1}{x}\right)$ is equal to _______.

(Q.16 - Q.20) Answer the following questions.

16. If the points $A(2, 3, 4)$, $B(-1, 2, -3)$ and $C(-4, 1, -10)$ are collinear, then find the ratio in which C divides AB.

17. Find $\lim_{n \to \infty} \frac{n!}{(n+1)! - n!}$.

OR

Evaluate : $\lim_{x \to 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{1+x}$

18. Write the negation of the following statement :
All mathematicians are man.

19. What is the locus of a point for which $x = 0$, $z = 0$?

20. Differentiate $(x^2 - 3x + 2)(x + 2)$ w.r.t. x.

SECTION-B

21. Find $\lim_{x \to 0} \frac{x(\cos x + \cos 2x)}{\sin x}$.

OR

Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$.

22. Let c denote contradiction and p be any statement. Then prove that $p \vee c \equiv p$.

23. Find the derivative of $\frac{2x^4 + x}{3x - 5}$ w.r.t. x.

24. Determine the point in xy-plane which is equidistant from the three points (2, 0, 3), (0, 3, 2) and (0, 0, 1).

OR

Find the points of trisection of the line segment joining the points (2, –2, 7) and (5, 1, –5).

25. Write the component statements of the following compound statements and check whether the compound statement is true or false :
(i) 125 is a multiple of 7 or 8.
(ii) Mumbai is the capital of Gujarat or Maharashtra.

26. Find the ratio in which the line joining the points (4, 4, – 10) and (–2, 2, 4) is divided by the xy-plane.

SECTION-C

27. Are the following statements negation of each other?
(i) "x is not a rational number."
"x is not a irrational number."
(ii) "x is not a real number."
"x is not an imaginary number."

28. Using the method of first principle find the derivative of $f(x) = \frac{2x + 7}{x + 2}$.

29. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{2^{-\cos x} - 1}{x\left(x - \frac{\pi}{2}\right)}$.

OR

Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$.

30. Verify that:
$(-1, 2, 1), (1, -2, 5), (4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

OR

Determine the values of a and b so that the points $(a, b, 3), (2, 0, -1)$ and $(1, -1, -3)$ are collinear.

31. If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a, b and c.

32. Check the validity of the following statements :

(i) Square of an integer is positive or negative.

(ii) If x and y are odd integers, then xy is an odd integer.

SECTION - D

33. For any three statements p, q, r prove that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

34. If α and β be the roots of equation $ax^2 + bx + c = 0$, then find $\lim\limits_{x\to\frac{1}{\alpha}} \sqrt{\dfrac{1-\cos(cx^2+bx+a)}{2(1-\alpha x)^2}}$.

OR

Evaluate : $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x - x^2}{x^2}$

35. If $y = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ... + \dfrac{x^n}{n!}$, then show that $\dfrac{dy}{dx} - y + \dfrac{x^n}{n!} = 0$.

36. The mid-points of the sides of a triangle are $(1, 5, -1)$, $(0, 4, -2)$ and $(2, 3, 4)$. Find its vertices.

OR

Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

SOLUTIONS

1. **(d):** Since A is the foot of perpendicular from P on xy-plane, so its z-coordinate is zero. Hence, coordinates of A is $(3, 4, 0)$. Similarly, we can find the coordinates of $B(0, 4, 5)$ and $C(3, 0, 5)$.

2. **(d):** We have, $\lim\limits_{k\to\infty}\left(\dfrac{1^3+2^3+...+k^3}{k^4}\right)$

$$= \lim_{k\to\infty}\frac{[k(k+1)/2]^2}{k^4} = \lim_{k\to\infty}\frac{\left(1+\frac{1}{k}\right)^2}{4} = \frac{1}{4}$$

3. **(a) :** 5 is less than 7. It is true and hence it is a statement.

4. **(a) :** The conditional statement of given statement is "If you take the dinner, then you will get a sweet dish".

5. **(b):** Given limit can be written as,

$$\lim_{x\to\frac{\pi}{4}}\frac{1}{\left(\frac{\pi}{4}-x\right)}\left(\frac{1-\tan x}{1+\tan x}\right) = \lim_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\frac{\pi}{4}-x} = 1$$

6. **(a) :** Length of edges of the parallelepiped are $5 - 2$, $9 - 3$, $7 - 5$ *i.e.*, 3, 6, 2.

$\therefore$ Length of diagonal is $\sqrt{3^2+6^2+2^2} = 7$ units.

7. **(a) :** 19 is a real number or a positive integer or both. So the statement is "inclusive or".

8. **(c) :** We have, $y = x^2 + \sin x + 1/x^2$

$$\therefore \frac{dy}{dx} = 2x + \cos x + (-2)x^{-3} = 2x + \cos x - \frac{2}{x^3}$$

9. **(d):** $x = \dfrac{1\times 1 + 4\times 2}{1+2} = 3$, $y = \dfrac{1\times 3 + 2\times 3}{1+2} = 3$

and $z = \dfrac{1\times 4 + 2\times 1}{1+2} = 2$. $\therefore$ Point is $(3, 3, 2)$.

10. **(b):** As $f'(x) = x\cos x + \sin x$

So, $f'\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}\cos\dfrac{\pi}{2} + \sin\dfrac{\pi}{2} = 1$

11. The three planes determine a rectangular parallelepiped which has three pairs of rectangular faces.

12. We have, $\lim\limits_{x\to 3}\dfrac{x^n - 3^n}{x-3} = n(3)^{n-1}$

Therefore, $n(3)^{n-1} = 108 = 4(27) = 4(3)^{4-1}$

On comparing, we get $n = 4$

OR

We have, $f(x) = x - [x]$ ...(i)

Differentiating (i) w.r.t. x, we get

$f'(x) = 1 - 0 = 1$ [$\because$ $[\cdot]$ is a constant function]

$\therefore$ $f'\left(\dfrac{1}{2}\right) = 1$

13. Perpendicular distance of the point $(6, 5, 8)$ from z-axis is $\sqrt{6^2+5^2+0^2} = \sqrt{36+25} = \sqrt{61}$ units.

14. The given sentence is false, so it is a statement.

OR

The quantifier in the given statement is "For all".

15. Since $\lim\limits_{x\to 0} x = 0$ and $-1 \le \sin\dfrac{1}{x} \le 1$

$\therefore$ By Sandwich theorem, we have $\lim\limits_{x\to 0} x\sin\dfrac{1}{x} = 0$

16. Given, $A \equiv (2, 3, 4)$, $B \equiv (-1, 2, -3)$, $C \equiv (-4, 1, -10)$
Let C divides AB in the ratio $k : 1$, then

$$C \equiv \left(\frac{-k+2}{k+1}, \frac{2k+3}{k+1}, \frac{-3k+4}{k+1}\right)$$

$$\therefore \quad \frac{-k+2}{k+1} = -4 \Rightarrow 3k = -6 \Rightarrow k = -2$$

Hence, C divides AB externally in the ratio 2 : 1.

17. $$\lim_{n\to\infty} \frac{n!}{(n+1)!-n!} = \lim_{n\to\infty} \frac{\frac{1}{n+1}}{1-\frac{1}{n+1}} = \frac{0}{1-0} = 0$$

OR

Here, $$\lim_{x\to 0} \frac{\sqrt{1+x}+\sqrt{1-x}}{1+x} = \frac{1+1}{1} = 2$$

18. The negation of the given statement is :
Some mathematicians are not man.

19. Locus of the point for which $x = 0$, $z = 0$ is y-axis.

20. Clearly, $\frac{d}{dx}\{(x^2-3x+2)(x+2)\}$

$$= \frac{d}{dx}(x^3 - x^2 - 4x + 4) = 3x^2 - 2x - 4$$

21. $$\lim_{x\to 0} \frac{x(\cos x + \cos 2x)}{\sin x} = \lim_{x\to 0} \frac{x(\cos x + \cos 2x)}{\frac{\sin x}{x}\cdot x}$$

[Here factor ($\cos x$ + $\cos 2x$), does not tends to zero, hence it is not necessary to simplify it]

$$= \lim_{x\to 0} \frac{\cos x + \cos 2x}{\frac{\sin x}{x}} = \frac{1+1}{1} = 2$$

OR

We have, $$\lim_{x\to 0} \frac{a^x - b^x}{x} = \lim_{x\to 0} \frac{(a^x-1)-(b^x-1)}{x}$$

$$= \lim_{x\to 0} \frac{a^x-1}{x} - \lim_{x\to 0} \frac{b^x-1}{x} = \log_e a - \log_e b = \log_e\left(\frac{a}{b}\right)$$

22. Truth table for $p \vee c$ is :

p	c	$p \vee c$
T	F	T
F	F	F

Hence from truth table, $p \vee c \equiv p$.

23. Let $y = \frac{2x^4 + x}{3x - 5}$

$$\therefore \quad \frac{dy}{dx} = \frac{(3x-5)\frac{d}{dx}(2x^4+x) - (2x^4+x)\frac{d}{dx}(3x-5)}{(3x-5)^2}$$

$$= \frac{(3x-5)(8x^3+1)-(2x^4+x)(3)}{(3x-5)^2} = \frac{18x^4 - 40x^3 - 5}{(3x-5)^2}$$

24. Let $P(x, y, 0)$ be the point in xy-plane which is equidistant from $A(2, 0, 3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$.
Then, $AP = BP = CP \Rightarrow AP^2 = BP^2 = CP^2$.
Now, $AP^2 = CP^2$

$$\Rightarrow \quad (x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 = x^2 + y^2 + 1 \Rightarrow x = 3$$

Again, $BP^2 = CP^2$

$$\Rightarrow \quad (x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$$

$$\Rightarrow \quad x^2 + y^2 + 9 - 6y + 4 = x^2 + y^2 + 1 \Rightarrow \quad y = 2$$

Hence, the required point is $P(x, y, 0) = P(3, 2, 0)$.

OR

Let P and Q be the points of trisection.
Then, $AP = PQ = QB$
$\therefore$ P divides AB internally in the ratio 1 : 2.

$$\therefore P \equiv \left(\frac{1\times 5 + 2\times 2}{2+1}, \frac{1\times 1 + 2\times(-2)}{2+1}, \frac{1\times(-5)+2\times 7}{2+1}\right)$$

or $P \equiv (3, -1, 3)$. Now, Q is the mid-point of PB

$$\therefore \quad Q \equiv \left(\frac{3+5}{2}, \frac{-1+1}{2}, \frac{3-5}{2}\right) \equiv (4, 0, -1)$$

25. (i) The component statements of the given statement are
p : 125 is a multiple of 7.
q : 125 is a multiple of 8.
We observe that both p and q are false statements. Therefore, the compound statement is also false.

(ii) The component statements of the given statement are
p : Mumbai is the capital of Gujarat.
q : Mumbai is the capital of Maharashtra.
We observe that p is false and q is true. Therefore, the compound statement is true.

26. Let $A \equiv (4, 4, -10)$, $B \equiv (-2, 2, 4)$
Let the line joining A and B be divided by the xy-plane at point P in the ratio $\lambda : 1$.

Then, $$P \equiv \left(\frac{-2\lambda+4}{\lambda+1}, \frac{2\lambda+4}{\lambda+1}, \frac{4\lambda-10}{\lambda+1}\right)$$

Since P lies on the xy-plane, therefore z-coordinate of P will be zero.

$$\therefore \quad \frac{4\lambda - 10}{\lambda+1} = 0 \Rightarrow \lambda = \frac{5}{2}$$

$\therefore$ Required ratio is 5 : 2 (internal).

27. (i) Let p : x is not a rational number.
and q : x is not an irrational number.
Then $\sim p$: x is a rational number.
and $\sim q$: x is an irrational number.

If x is a rational number, then x is not an irrational number. Therefore, $\sim p = q$.

Again if x is an irrational number, then x is not a rational number. Therefore, $\sim q = p$.

Thus, p and q are negation of each other.

(ii) Let $p : x$ is not a real number.

and $q : x$ is not an imaginary number.

Then $\sim p : x$ is a real number.

and $\sim q : x$ is an imaginary number.

If x is an imaginary number, then it is definitely not a real number. Therefore $\sim q = p$.

Again if x is a real number, then x is definitely not an imaginary number. Therefore, $\sim p = q$.

Thus p and q are negation of each other.

28. *Refer to answer 101, page no. 296 of MTG CBSE Champion Mathematics, Class-11*

29. Put $x = \frac{\pi}{2} + h$. As $x \to \frac{\pi}{2}, h \to 0$

$$\text{Now, } \lim_{x \to \frac{\pi}{2}} \frac{2^{-\cos x} - 1}{x\left(x - \frac{\pi}{2}\right)} = \lim_{h \to 0} \frac{2^{-\cos\left(\frac{\pi}{2}+h\right)} - 1}{\left(\frac{\pi}{2}+h\right)\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)}$$

$$= \lim_{h \to 0} \frac{2^{\sin h} - 1}{\left(\frac{\pi}{2}+h\right)h} = \lim_{h \to 0} \frac{2^{\sin h} - 1}{\sin h} \cdot \frac{\sin h}{h} \cdot \frac{1}{\left(\frac{\pi}{2}+h\right)}$$

$$= \log_e 2 \times 1 \times \frac{1}{\pi/2} = \frac{2}{\pi} \log_e 2$$

OR

$$\lim_{x \to \pi/2} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x} = \lim_{x \to \pi/2} a^{\cos x}\left[\frac{a^{\cot x - \cos x} - 1}{\cot x - \cos x}\right] \quad ...(i)$$

Put $\cot x - \cos x = t$

$$\text{Now, } x \to \frac{\pi}{2} \Rightarrow \cot\frac{\pi}{2} - \cos\frac{\pi}{2} = t \;\Rightarrow\; t \to 0$$

$$\therefore \quad \text{(i) becomes } \lim_{t \to 0} \frac{a^t - 1}{t} = \log_e a$$

30. *Refer to answer 39, page no. 271 of MTG CBSE Champion Mathematics, Class-11*

OR

Suppose the given points are $P(a, b, 3)$, $Q(2, 0, -1)$ and $R(1, -1, -3)$.

Let Q divides the line segment PR in the ratio $k : 1$.

$$\text{Then coordinates of Q are } \left(\frac{k+a}{k+1}, \frac{-k+b}{k+1}, \frac{-3k+3}{k+1}\right)$$

But coordinates of Q are $(2, 0, -1)$

$$\therefore \quad (2, 0, -1) \equiv \left(\frac{k+a}{k+1}, \frac{-k+b}{k+1}, \frac{-3k+3}{k+1}\right)$$

$$\Rightarrow \quad \frac{k+a}{k+1} = 2, \frac{-k+b}{k+1} = 0, \frac{-3k+3}{k+1} = -1$$

$$\text{Now, } \frac{-3k+3}{k+1} = -1 \Rightarrow -3k + 3 = -k - 1 \Rightarrow k = 2$$

$$\therefore \quad \frac{k+a}{k+1} = 2 \Rightarrow \frac{2+a}{3} = 2 \;\Rightarrow\; a = 6 - 2 = 4$$

$$\text{and } \frac{-k+b}{k+1} = 0 \Rightarrow \frac{-2+b}{3} = 0 \Rightarrow b = 2.$$

31. *Refer to answer 68, page no. 275 of MTG CBSE Champion Mathematics, Class-11*

32. **(i)** The given statement is a compound statement with "OR" whose component statements are:

p : Square of an integer is positive.

q : Square of an integer is negative.

Let us assume that p is false *i.e.,* square of an integer is not positive. Then, for any integer x, we have

$x^2 \ngeq 0$

$\Rightarrow \quad x^2 < 0 \Rightarrow q$ is true.

Thus, if we assume that p is false, then q is true.

Hence, "p or q" is a valid statement. In other words, the given statement is true.

(ii) Let p and q be the statements given by

$p : x$ and y are odd integers

$q : xy$ is an odd integer.

Let us assume that q is false.

$\Rightarrow \quad xy$ is an even integer

$\Rightarrow \quad$ either x is even or y is even or both x and y are even

$\Rightarrow \quad p$ is not true.

Thus, q is false $\Rightarrow$ p is false

Hence, "If p, then q" is a true statement.

33. The truth table for $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ is :

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

From the truth table it follows that

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

34. Since α, β are the roots of equation $ax^2 + bx + c = 0$, therefore roots of equation $cx^2 + bx + a = 0$ will be $1/\alpha$ and $1/\beta$.

$\therefore \quad cx^2 + bx + a = c(x - 1/\alpha)(x - 1/\beta)$

Required limit $= \lim\limits_{x \to \frac{1}{\alpha}} \sqrt{\dfrac{1 - \cos\{c(x - 1/\alpha)(x - 1/\beta)\}}{2(1 - \alpha x)^2}}$

$$= \lim_{x \to \frac{1}{\alpha}} \sqrt{\frac{2\sin^2\{(c/2)(x - 1/\alpha)(x - 1/\beta)\}}{2(1 - \alpha x)^2}}$$

$$= \lim_{x \to \frac{1}{\alpha}} \left| \frac{\sin\left\{\frac{c}{2}\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\right\}}{1 - \alpha x} \right| \qquad \left[\because \sqrt{x^2} = |x|\right]$$

$$= \lim_{x \to \frac{1}{\alpha}} \left| \frac{\sin\left\{\frac{c}{2}\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)\right\}}{\frac{c}{2}\left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)} \cdot \frac{c(\alpha x - 1)(\beta x - 1)}{2\alpha\beta(1 - \alpha x)} \right|$$

$$= \left| \frac{c}{2\alpha\beta}\left(\frac{\beta}{\alpha} - 1\right) \right| = \left| \frac{c}{2\alpha}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right) \right|$$

OR

Refer to answer 68, page no. 293 of MTG CBSE Champion Mathematics, Class-11

35. We have, $y = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^n}{n!}$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(1) + \frac{d}{dx}\left(\frac{x}{1!}\right) + \frac{d}{dx}\left(\frac{x^2}{2!}\right) + \ldots + \frac{d}{dx}\left(\frac{x^n}{n!}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = 0 + \frac{1}{1!} + \frac{1}{2!}(2x) + \frac{1}{3!}(3x^2) + \ldots + \frac{1}{n!}(nx^{n-1})$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left\{1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}\right\} - \frac{x^n}{n!}$$

$$\Rightarrow \quad \frac{dy}{dx} = y - \frac{x^n}{n!} \quad \Rightarrow \quad \frac{dy}{dx} - y + \frac{x^n}{n!} = 0$$

36. Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of the given triangle, and let $D(1,5,-1)$, $E(0,4,-2)$ and $F(2,3,4)$ be the mid-points of the sides BC, CA and AB respectively.

As, D is the mid-point of BC.

$$\therefore \quad \frac{x_2 + x_3}{2} = 1, \frac{y_2 + y_3}{2} = 5, \frac{z_2 + z_3}{2} = -1$$

$$\Rightarrow \quad x_2 + x_3 = 2, y_2 + y_3 = 10, z_2 + z_3 = -2 \qquad \ldots\text{(i)}$$

Similarly, E and F are the mid-points of CA and AB respectively.

$\therefore \quad x_1 + x_3 = 0, y_1 + y_3 = 8, z_1 + z_3 = -4 \qquad \ldots$(ii)

and $x_1 + x_2 = 4, y_1 + y_2 = 6, z_1 + z_2 = 8 \qquad \ldots$(iii)

Adding first three equations in (i), (ii) and (iii), we get

$2(x_1 + x_2 + x_3) = 6 \Rightarrow x_1 + x_2 + x_3 = 3 \qquad \ldots$(iv)

Solving first equations in (i), (ii) and (iii) with (iv) we get, $x_1 = 1, x_2 = 3, x_3 = -1$

Adding second equations in (i), (ii) and (iii), we get

$2(y_1 + y_2 + y_3) = 10 + 8 + 6 \Rightarrow y_1 + y_2 + y_3 = 12 \qquad \ldots$(v)

Solving second equations in (i), (ii) and (iii) with (v) we get, $y_1 = 2, y_2 = 4, y_3 = 6$

Adding last equations in (i), (ii) and (iii), we get

$2(z_1 + z_2 + z_3) = -2 - 4 + 8 \Rightarrow z_1 + z_2 + z_3 = 1 \qquad \ldots$(vi)

Solving last equations in (i), (ii) and (iii) with (vi) we get, $z_1 = 3, z_2 = 5, z_3 = -7$

Thus, the vertices of the triangle are $A(1,2,3)$, $B(3,4,5)$ and $C(-1, 6, -7)$.

OR

Refer to answer 73, page no. 277 of MTG CBSE Champion Mathematics, Class-11

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How to Study and Crack Competitive Exam:

12 Study Hacks You Must Know

India has an extremely competitive environment. Numerous competitive exams are conducted every year for admission to undergraduate and postgraduate professional courses as well as for securing services in the government. Every year lakhs of students appeared for these exams but only a handful of them have cleared it. As the competition is very tough, so there is no space for mistakes and a candidate has to be thoroughly prepared for the exams. So, we've rounded up 12 tips that will help you to get the most out of your studying hours.

① **Devise a Routine :** Before you start preparing for any competitive exam, you must design a timetable and set yourself with the clear daily, weekly and monthly targets. Ensure your plan is not aggressive that you feel overburdened. Make short study sessions, they are most effective.

② **Create a Good Study Environment :** Make sure your study place is pleasant. It is the key element for effective study. A good study environment allows you to maintain your concentration and maximize your learning efficiency.

③ **Avoid Rote Learning and Focus on Basic Concepts :** Focus on understanding the concepts throughout your preparation. Do not adopt rote learning techniques. Because exams like JEE Main, JEE Advanced and other PETs are extremely complex and check your analytical skills. You need to understand them and know how they are interlinked.

④ **Get Creative and Colourful :** Draw a clear diagram or layout of what you need to learn as per your understanding. Make it visually creative and colourful by highlighting or underlining with different colours. List the dates, formulas, mnemonics and tit-bits of important information that you find difficult to learn. Stick them somewhere on the wall. Seeing them in front of your eyes all the time will help to feed them into the memory very easily.

⑤ **Past Papers – Do a lot! :** Previous years papers really make a lot of difference in your preparation. If you are preparing for exams like JEE Main, solving previous years' question papers definitely helps you.

⑥ **Rest, Eat, Exercise :** As we mentioned above, regular breaks are very necessary throughout your study session. You need to take proper rest and have a healthy diet and don't feel guilty about spending time for some recreational activities. This will help you to boost back with full energy.

⑦ **Boost your Short-Term Memory :** When your exam date is too close then learning new theories from the start is absolutely not preferable. Rather strengthen your short-term memory by solving previous years papers.

⑧ **Start with Easiest :** Get the exam rhythm, read all the questions carefully. Divide your time for each question. Then start with the easiest question. Easy questions won't take much time and you will get enough time to solve complex questions.

⑨ **Don't go out of the Box while Answering :** Don't mark an answer if you are not sure about it. This will result as negative marking.

⑩ **If Stuck, Move on :** Just calm down if you stuck on any question and leave it aside for last hour. Answer the next question and regain confidence. Once done with all known questions come back to the stuck ones with a different perspective and you will definitely solve it.

⑪ **Stay Hydrated Always :** Keep drinking plenty of water, to avoid drowsiness and headaches. And also, at times when you get nervous.

⑫ **Never Leave Early :** It feels great when you finish your answer sheets well before time. But don't leave the hall till the final bell rings. Take the opportunity to recheck and revise the entire answer sheet again and again. At this point, some students make the mistake of leaving the exam room. Once you leave it, it's too late – you can't go back in for marking unmarked questions. So, even if you finish early, use that extra time to read your answer sheet and make sure that you've answered all the questions to the best of your abilities.

"Always keep in mind that a disturbed and a non-planned study can lead to loss of marks in the examination." Therefore, by following these 12 tips listed above, we hope that your next exam will be an enjoyable experience with a successful outcome.

All the Best! ☝

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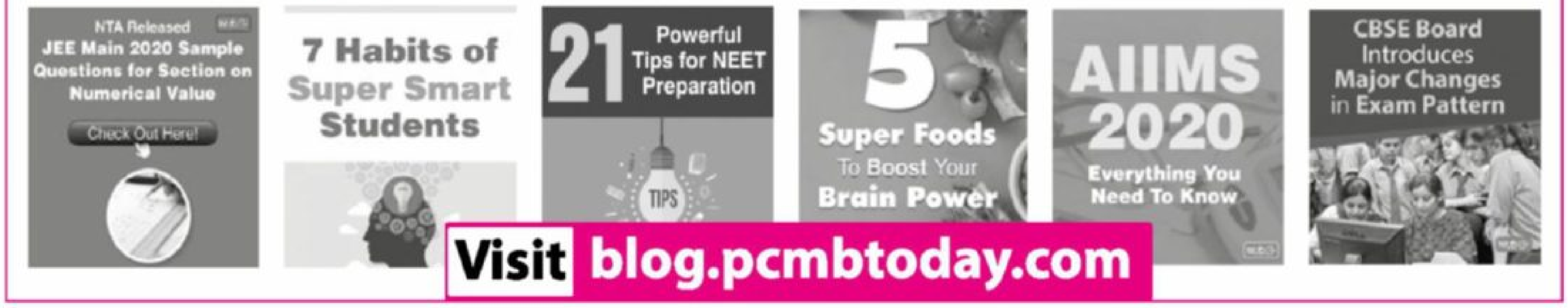
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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 7 : Limits and Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

1. The value of $\lim\limits_{x \to -\infty} (\sqrt{x^2 - x + 1} + x)$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) –1

2. The value of $\lim\limits_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

3. The value of $\lim\limits_{x \to 0} \left\{\frac{a^x + b^x + c^x}{3}\right\}^{1/x}$ is

(a) abc (b) $(abc)^{1/3}$

(c) $\frac{1}{3}abc$ (d) none of these

4. If $y = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is

(a) $\frac{1 - \sqrt{3}}{2}$ (b) 0

(c) $\frac{\sqrt{3} - 1}{2}$ (d) none of these

5. Let $f(x) = \frac{\log_e (x^2 + e^x)}{\log_e (x^4 + e^{2x})}$. If $\lim\limits_{x \to \infty} f(x) = l$ and $\lim\limits_{x \to -\infty} f(x) = m$, then

(a) $l = m$ (b) $l = 2m$

(c) $2l = m$ (d) $l + m = 0$

6. If $y = f\left(\frac{2x - 1}{x^2 + 1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx}$ is equal to

(a) $\sin\left(\frac{2x - 1}{x^2 + 1}\right)^2 \left\{\frac{x^2 + 2x + 2}{(x^2 + 1)^2}\right\}$

(b) $\sin\left(\frac{2x - 1}{x^2 + 1}\right)^2 \left\{\frac{2 + 2x - 2x^2}{(x^2 + 1)^2}\right\}$

(c) $\sin\left(\frac{2x - 1}{x^2 + 1}\right)^2 \left\{\frac{2 + 2x - x^2}{(x^2 + 1)}\right\}$

(d) none of these

One or More Than One Option(s) Correct Type

7. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2y - 1}$ (b) $\frac{x}{x + 2y}$

(c) $\frac{1}{\sqrt{1 + 4x}}$ (d) $\frac{y}{2x + y}$

8. $\lim\limits_{x \to 0} \left(\sum_{r=1}^{n} r^{\operatorname{cosec}^2 x}\right)^{\sin^2 x} =$

(a) 0 (b) ∞ (c) n (d) $\frac{1}{n}$

9. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct?

(a) $\lim\limits_{x \to 1} f(x)$ exists $\Rightarrow a = -2$

(b) $\lim\limits_{x \to -2} f(x)$ exists $\Rightarrow a = 13$

(c) $\lim\limits_{x \to 1} f(x) = \frac{4}{3}$ (d) $\lim\limits_{x \to -2} f(x) = -\frac{1}{3}$

10. The value of a for which

$$\lim_{x\to 0}\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{a^2}\right)\log_e\left\{1+\frac{x^2}{2}\right\}}=8, \text{ is}$$

(a) -2 (b) -1 (c) 1 (d) 2

11. If $\lim\limits_{x\to 0}(\cos x+a\sin bx)^{1/x}=e^2$, then the values of a and b are

(a) $a=1, b=2$ (b) $a=2, b=\frac{1}{2}$

(c) $a=2\sqrt{2}, b=\frac{1}{\sqrt{2}}$ (d) $a=4, b=2$

12. If $m, n\in N$, $\lim\limits_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$ is equal to

(a) 1, if $n=m$ (b) 0, if $n>m$

(c) ∞, if $n<m$ (d) n/m, if $n<m$

13. Let $f(x)=\frac{x^2-9x+20}{x-[x]}$, (where $[\cdot]$ denotes the greatest integer function), then

(a) $\lim\limits_{x\to 5^-}f(x)=0$ (b) $\lim\limits_{x\to 5^+}f(x)=1$

(c) $\lim\limits_{x\to 5}f(x)$ does not exist

(d) none of these

Comprehension Type

Let $a_1>a_2>a_3>....>a_n>1$ and $p_1>p_2>p_3>....>p_n>0$ such that $p_1+p_2+p_3+....+p_n=1$. Also, $F(x)=(p_1a_1^x+p_2a_2^x+....+p_na_n^x)^{1/x}$.

14. $\lim\limits_{x\to 0^+}F(x)$ equals

(a) $p_1\ln a_1+p_2\ln a_2+....+p_n\ln a_n$

(b) $a_1^{p_1}+a_2^{p_2}+....+a_n^{p_n}$

(c) $a_1^{p_1}\cdot a_2^{p_2}....a_n^{p_n}$ (d) $\sum\limits_{r=1}^{n}a_rp_r$

15. $\lim\limits_{x\to\infty}F(x)$ equals

(a) $\ln a_1$ (b) e^{a_n} (c) a_1 (d) a_n

Matrix Match Type

16. Match the columns:

	Column-I		Column-II
P.	If $f(x)=\left(\frac{\lvert x\rvert}{\lvert x\rvert+2}\right)^{-x}$, then	1.	$\lim\limits_{x\to\infty}f(x)=e^2$
Q.	If $f(x)=\frac{(1+x)^{1/x}-e}{x}$, then	2.	$\lim\limits_{x\to 0}f(x)=e^2$
R.	If $f(x)=\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$, then	3.	$\lim\limits_{x\to-\infty}f(x)=e^{-2}$
		4.	$\lim\limits_{x\to 0}f(x)=-\frac{e}{2}$
		5.	$\lim\limits_{x\to 0}f(x)<-1$

	P	Q	R
(a)	1, 3	4, 5	2
(b)	2, 3	4, 5	1
(c)	2, 3	1, 5	2
(d)	3, 4	1, 5	2

Numerical Value Type

17. The value of $\lim\limits_{x\to 0}27e^{-2}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$ must be

18. Let $\lim\limits_{x\to 1}\frac{x^a-ax+a-1}{(x-1)^2}=f(a)$. Then the value of $f(4)$ is

19. If $L=\lim\limits_{x\to 0}\frac{e^{-x^2/2}-\cos x}{x^3\sin x}$, then the value of $3L$ is

20. If $y=\sqrt{\left(\frac{1+\cos 2\theta}{1-\cos 2\theta}\right)}$, then $\frac{dy}{d\theta}$ at $\theta=\frac{3\pi}{4}$ is

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

FOCUS

Between
6th to 11th
January

JEE MAIN 2020

with Numerical Value Type Questions

PRACTICE PAPER

1. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(a) 0 (b) 1 (c) 2 (d) ∞

2. $\lim_{x\to\pi/2} \frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3} =$

(a) 0 (b) 1/32 (c) ∞ (d) 1/8

3. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

4. The range of the function $F(x) = {}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 4, 5, 6\}$

(c) $\{1, 2, 3\}$ (d) $\{1, 2, 3, 4, 5\}$

5. If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

6. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex further, assume that the origin, z_1 and z_2 form an equilateral triangle, then

(a) $a^2 = 2b$ (b) $a^2 = 3b$

(c) $a^2 = 4b$ (d) $a^2 = b$

7. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(a) $\frac{2\pi}{3}-4-4\sqrt{3}$ (b) $4\sqrt{3}-4$

(c) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) $\pi - 4$

8. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(a) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\frac{dy}{dx}$

(c) $x^2 = y^2 + xy\frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\frac{dy}{dx}$

9. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \ldots \infty$ is

(a) 1 (b) 2 (c) 3/2 (d) 4

10. If $1, \omega, \omega^2$ are the cube roots of unity,

then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0

11. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is

(a) 1594 (b) 792 (c) 924 (d) 2924

12. A straight line through the point $M(3, 4)$ is such that its intercept between the axes is bisected at M. Its equation is

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$

(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

13. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1}=\frac{y-2}{5}=\frac{z-3}{4}$ also contains the point

(a) (0, –3, 1) (b) (0, 7, 10)
(c) (0, 7, –10) (d) (0, 3, 1)

14. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

(a) 5040 (b) 6210 (c) 385 (d) 1110

15. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ then $\vec{a}+2\vec{b}+6\vec{c}$ equals (λ being some non-zero scalar)

(a) $\lambda\vec{c}$ (b) $\lambda\vec{b}$ (c) $\lambda\vec{a}$ (d) 0

16. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \leftrightarrow q)$ (b) $p \to (p \to \sim q)$
(c) $p \to (p \vee \sim q)$ (d) $p \to (p \wedge q)$

17. The area of the region bounded by the curves $y = |x-1|$ and $y = 3 - |x|$ is

(a) 3 sq. units (b) 4 sq. units
(c) 6 sq. units (d) 2 sq. units

18. The sum of first 9 terms of the series

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots \text{ is}$$

(a) 142 (b) 192 (c) 71 (d) 96

19. In an experiment with 15 observations on x, the following results were available.

$$\sum x^2 = 2830, \sum x = 170$$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(a) 188.66 (b) 177.33 (c) 8.33 (d) 78

20. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

(a) $-\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $-\frac{4}{3}$ (d) $\frac{3}{4}$

Numerical Value Type

21. Let $A=\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

22. If $f(x+y) = f(x) \cdot f(y)$ $\forall$ x, y and $f(5) = 2$, $f'(0) = 3$, then the value of $f'(5)$ is

23. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30°$. The absolute value of the difference between the areas of these triangles is

24. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

25. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and $F = \{X < 4\}$, $P(E \cup F)$ is

SOLUTIONS

1. (b) : We have, $|z-1| = |z+1|$

$\Rightarrow |z-1|^2 = |z+1|^2$

$\Rightarrow (z-1)(\bar{z}-1) = (z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z} - z - \bar{z} + 1 = z\bar{z} + z + \bar{z} + 1$

$\Rightarrow z + \bar{z} = 0$ (z being purely imaginary)

Thus, $x = 0$

Again, $|z-1|^2 = |z-i|^2$

$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow 1 + y^2 = (y-1)^2$ $(\because\ x = 0)$

$\Rightarrow 1 + y^2 = y^2 - 2y + 1$

$\therefore\ y = 0$

Thus, (0, 0) is the desired point.

Monthly Test Drive-7 CLASS XII ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(b)	**4.**	(c)
5.	(b)	**6.**	(a)	**7.**	(b,d)	**8.**	(a,b,c)
9.	(a,b)	**10.**	(a,c,d)	**11.**	(a,b,c,d)	**12.**	(b,d)
13.	(a,c)	**14.**	(c)	**15.**	(d)	**16.**	(b)
17.	(20)	**18.**	(5050)	**19.**	(4)	**20.**	(6)

2. (b) : $\displaystyle\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{4\cdot\left(\frac{\pi-2x}{4}\right)(\pi-2x)^2}$

$$=\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4\cdot\left(\frac{\pi}{4}-\frac{x}{2}\right)}\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{(\pi-2x)^2}$$

$$=\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4\cdot\left(\frac{\pi}{4}-\frac{x}{2}\right)}\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4^2\left(\frac{\pi-2x}{4}\right)^2}=\frac{1}{4}\times\frac{2}{16}=\frac{1}{32}$$

3. (d) : $f'(x)=\dfrac{1}{1+(\sin x+\cos x)^2}.(\cos x-\sin x)$

$\Rightarrow\quad f'(x)=\dfrac{\cos x-\sin x}{2+\sin 2x}$

If $f'(x) > 0$ then $f(x)$ is an increasing function.

For $-\dfrac{\pi}{2}<x<\dfrac{\pi}{4}, \cos x>\sin x$

Hence, $y = f(x)$ is increasing in $\left(-\dfrac{\pi}{2},\dfrac{\pi}{4}\right)$.

4. (c) : $F(x)$ to be defined for $x \in N$.

(i) $\therefore\ 7 - x > 0 \Rightarrow x < 7$

(ii) $x - 3 \geq 0 \Rightarrow x \geq 3$

(iii) $x - 3 \leq 7 - x \Rightarrow x \leq 5$

$\therefore$ From (i), (ii) and (iii), $x = 3, 4, 5$

$\therefore\ F(3) = {}^4P_0,\ F(4) = {}^3P_1,\ F(5) = {}^2P_2$

$\therefore$ {1, 2, 3} is required range.

5. (c) : Given, $\cos x+\sin x=\dfrac{1}{2}$

$\Rightarrow\ 1+\sin 2x=\dfrac{1}{4}$ (By squaring both sides)

$\Rightarrow\ \dfrac{2\tan x}{1+\tan^2 x}=\dfrac{-3}{4}$

$\Rightarrow\ 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore\ \tan x=\dfrac{-8\pm\sqrt{64-36}}{6}=\dfrac{-4\pm\sqrt{7}}{3}$

6. (b) : As z_1, z_2 are roots of $z^2 + az + b = 0$

$\therefore\ z_1 + z_2 = -a,\ z_1z_2 = b$

Again 0, z_1, z_2 are vertices of an equilateral triangle

$\therefore\ 0^2 + z_1^2 + z_2^2 = 0z_1 + z_1z_2 + z_20 = 0$

$\Rightarrow\ z_1^2 + z_2^2 = z_1z_2 \Rightarrow\ (z_1 + z_2)^2 = 3z_1z_2$

$\Rightarrow\ a^2 = 3b$

7. (c) : Let $I=\displaystyle\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$$=\int_0^{\pi}\sqrt{\left(1-2\sin\frac{x}{2}\right)^2}\,dx=\int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$$

$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$

$$=\left(x+4\cos\frac{x}{2}\right)\Bigg|_0^{\pi/3}+\left(-4\cos\frac{x}{2}-x\right)\Bigg|_{\pi/3}^{\pi}$$

$$=-\frac{\pi}{3}+8\cdot\frac{\sqrt{3}}{2}-4=4\sqrt{3}-4-\frac{\pi}{3}$$

8. (a) : General equation of all such circles is

$(x - h)^2 + (y - 0)^2 = h^2$ ....(i) where h is parameter

$\Rightarrow\ (x - h)^2 + y^2 = h^2$

Differentiating w.r.t. x, we get

$2(x-h)+2y\dfrac{dy}{dx}=0\ \Rightarrow\ h=x+y\dfrac{dy}{dx}$

Putting the value of h in (i), we get

$y^2=x^2+2xy\dfrac{dy}{dx}$

9. (b) : Let $S_\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+....\infty} = 2^{\lambda}$(say),

where $\lambda = \dfrac{1}{4}+\dfrac{2}{8}+\dfrac{3}{16}+\dfrac{4}{32}+\ ....\ \infty$ ...(i)

$\Rightarrow\ \dfrac{\lambda}{2} = 0 + \dfrac{1}{8}+\dfrac{2}{16}+\dfrac{3}{32}+\dfrac{4}{64}+\ ...\ \infty$ ...(ii)

Now, (i) – (ii) $\Rightarrow\ \dfrac{\lambda}{2}=\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{32}+\dfrac{1}{64}+...\infty$

$\Rightarrow\ \dfrac{\lambda}{2}=\dfrac{\frac{1}{4}}{1-\frac{1}{2}}=\dfrac{1}{4}\times\dfrac{2}{1}\ \Rightarrow\ \lambda=1$

So, $S_\infty = 2^1 = 2$

10. (d) : As ω is cube root of unity $\therefore\ \omega^3 = \omega^{3n} = 1$

$$\therefore\ \begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^n & \omega^{2n} & 1\\ \omega^{2n} & 1 & \omega^n\end{vmatrix}$$

$= (\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^n) = 0$

11. (c) : Consider $(a + b)^n = C_0\, a^n + C_1 a^{n-1}b + C_2 a^{n-2}b^2 + ... + C_n b^n$

Putting $a = b = 1$, we get

$2^n = C_0 + C_1 + C_2 + ... + C_n$

$\Rightarrow\ 2^n = 4096 = 2^{12} \Rightarrow\ n = 12$ (even)

Now, $(a + b)^n = (a + b)^{12}$

As $n = 12$ is even so coefficient of greatest term is

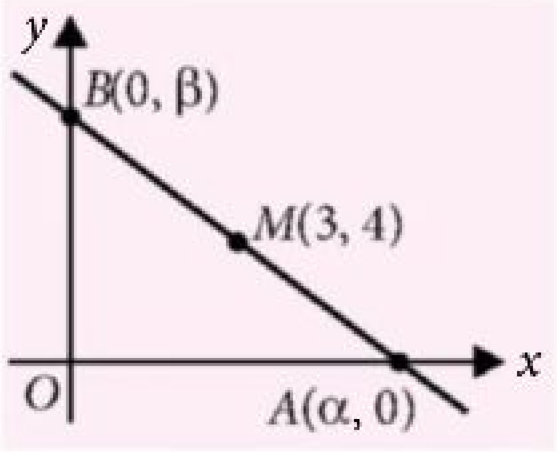
$${}^{n}C_{n/2} = {}^{12}C_{12/2} = {}^{12}C_6 = 924$$

12. (c) : The equation of line which meets the x-axis and y-axis at $A(\alpha, 0)$ $B(0, \beta)$ is given by

$$\frac{x}{\alpha} + \frac{y}{\beta} = 1,$$

where $\alpha = 2(3) = 6$

and $\beta = 2(4) = 8$.



$\therefore$ Required equation is $\frac{x}{6} + \frac{y}{8} = 1$

$\Rightarrow\ 4x + 3y = 24$

13. (b) : $A(3, 2, 0)$ and $B(1, 2, 3)$ lie in the plane.

$\Rightarrow\ \overrightarrow{AB} = 2\hat{i} + 0\hat{j} + (-3)\hat{k}$ also lie in the plane.

$\therefore$ Normal vector of plane $= (2\hat{i} - 3\hat{k}) \times (\hat{i} + 5\hat{j} + 4\hat{k})$

$= 15\hat{i} - 11\hat{j} + 10\hat{k}$

$\therefore$ Equation of plane is

$(\vec{r} - (3\hat{i} + 2\hat{j} + 0\hat{k})) \cdot (15\hat{i} - 11\hat{j} + 10\hat{k}) = 0$

$\Rightarrow\ 15x - 11y + 10z - 23 = 0$

14. (c) : A voter can vote for one candidate or two or three or four candidates.

$\therefore$ Required number of ways

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 385$$

15. (d) : As $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$.

$\therefore\ \vec{a} + 2\vec{b} = P\vec{c}$ ...(i)

and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ $\therefore\ \vec{b} + 3\vec{c} = Q\vec{a}$ ...(ii)

Now by (i) and (ii), we have

$\vec{a} - 6\vec{c} = P\vec{c} - 2Q\vec{a}$

$\Rightarrow\ \vec{a}\,(1 + 2Q) + \vec{c}\,(-6 - P) = 0$

$\Rightarrow\ 1 + 2Q = 0$ and $-P - 6 = 0$

$\therefore\ Q = -1/2,\ P = -6$

Putting these values either in (i) or in (ii), we get

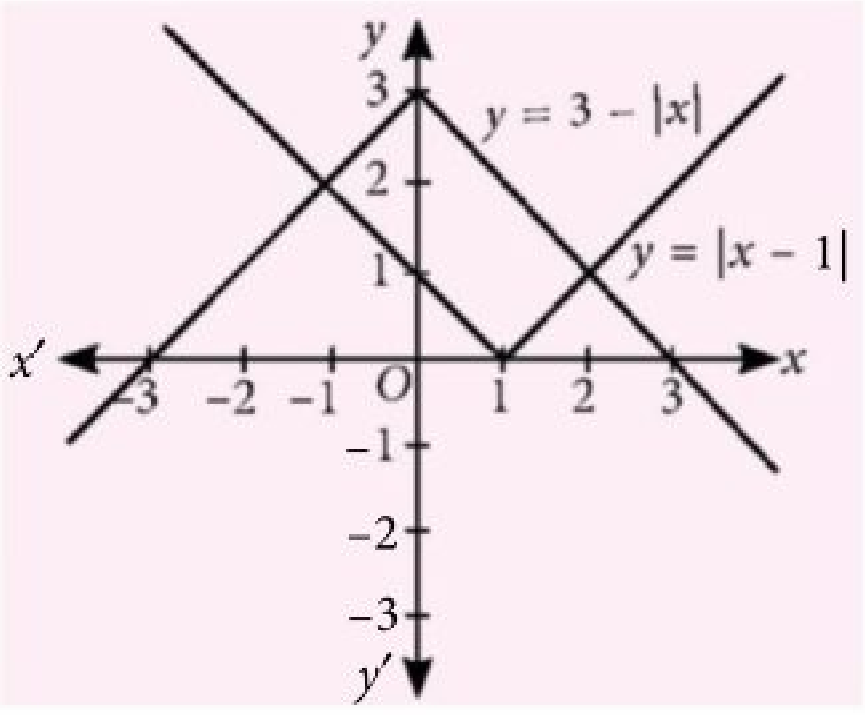
$$\vec{a} + 2\vec{b} + 6\vec{c} = 0$$

16. (c) : Let's simplify the statement

$p \rightarrow (q \rightarrow p) \equiv \sim p \vee (q \rightarrow p) \equiv \sim p \vee (\sim q \vee p)$

$\equiv \sim p \vee (p \vee \sim q) \equiv p \rightarrow (p \vee \sim q)$

17. (b) : Given, curves are $y = 3 - |x|$ and $y = |x - 1|$.



Required area

$$= \int_{-1}^{0} [(3+x)-(-x+1)]dx + \int_{0}^{1} [(3-x)-(-x+1)]dx + \int_{1}^{2} [(3-x)-(x-1)]\,dx$$

$$= \int_{-1}^{0} 2(1+x)dx + \int_{0}^{1} 2\,dx + \int_{1}^{2} (4-2x)\,dx = 4 \text{ sq. units}$$

18. (d) : The n^{th} term, t_n is

$$\frac{1^3 + 2^3 + ... + n^3}{1 + 3 + ... + (2n-1)} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4}$$

$$\therefore\ \sum_{n=1}^{9} t_n = \sum_{n=1}^{9} \frac{(n+1)^2}{4} = \frac{1}{4}\left\{\sum_{n=1}^{10} n^2 - 1\right\}$$

$$= \frac{1}{4}\left\{\frac{10 \cdot 11 \cdot 21}{6} - 1\right\} = \frac{1}{4}\{385 - 1\} = \frac{1}{4} \times 384 = 96$$

19. (d) : We have, $\sum x = 170$ and $\sum x^2 = 2830$

Increase in $\sum x = 10 \Rightarrow \sum x' = 170 + 10 = 180$

Increase in $\sum x^2 = 900 - 400 = 500$ then

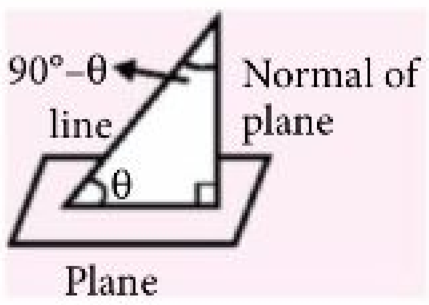
$\sum x'^2 = 2830 + 500 = 3330$

$$\therefore\ \sigma^2 = \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - (12)^2 = 78$$

20. (b) : Angle between the line and plane is complement to the angle between the line and normal to the plane.

$$\therefore\ \cos(90^\circ - \theta) = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow\ \frac{1}{3} = \frac{(1 \times 2 + 2 \times (-1)) + 2\sqrt{\lambda}}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + 1^2 + \lambda}}$$



$$\Rightarrow\ \lambda = \frac{5}{3}$$

21. (0.2) : $A^2=\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}\begin{bmatrix}5 & 5\alpha & \alpha\\ 0 & \alpha & 5\alpha\\ 0 & 0 & 5\end{bmatrix}$

$$\Rightarrow\quad A^2=\begin{bmatrix}25 & 25\alpha+5\alpha^2 & 5\alpha+5\alpha+25\alpha^2\\ 0 & \alpha^2 & 25\alpha+5\alpha^2\\ 0 & 0 & 25\end{bmatrix}$$

Given, $|A^2| = 25 \Rightarrow 625\alpha^2 = 25 \Rightarrow |\alpha|=\frac{1}{5}=0.2$

22. (6) : Given $f(x+y) = f(x)\,f(y)$

$\therefore\ f(0+0) = (f(0))^2$

$\Rightarrow f(0) = 0$ or $f(0) = 1$ but $f(0) \neq 0$

Now $f'(x)=\underset{h\to 0}{\text{Lt}}\dfrac{f(x+h)-f(x)}{h}$

$$\Rightarrow\ f'(x)=\underset{h\to 0}{\text{Lt}}\frac{f(x)f(h)-f(x)}{h}=f(x)\underset{h\to 0}{\text{Lt}}\frac{f(h)-1}{h}$$

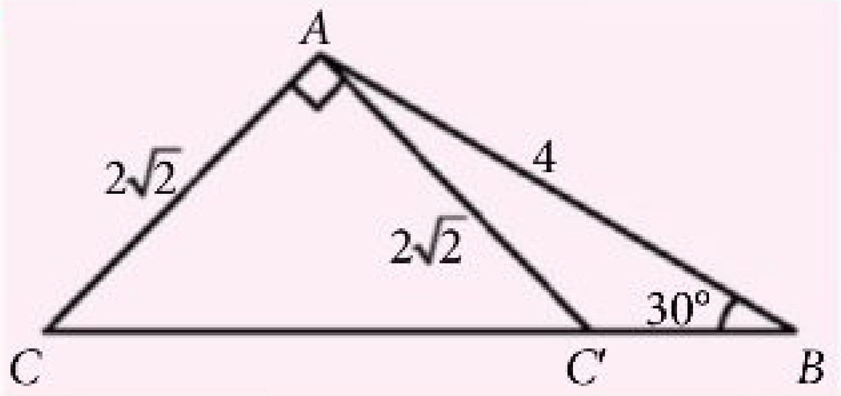
$$\therefore\ f'(0)=f(0)\underset{h\to 0}{\text{Lt}}\frac{f(h)-1}{h}$$

$$\Rightarrow\ 3=\underset{h\to 0}{\text{Lt}}\frac{f(h)-1}{h}\qquad (\because\ f(0)=1)$$

Now, $f'(x)=f(x)\underset{h\to 0}{\text{Lt}}\dfrac{f(h)-1}{h}$

$\therefore\ f'(5) = f(5)\times 3 = 2\times 3 = 6$

23. (4) :



Let the third side be a. From cosine rule, we have

$$(2\sqrt{2})^2=4^2+a^2-2\times 4a\cos 30^\circ$$

$$\Rightarrow\ 8=16+a^2-8a\frac{\sqrt{3}}{2}\ \Rightarrow\ a^2-4a\sqrt{3}+8=0$$

$$\text{Giving }\ a=\frac{4\sqrt{3}\pm\sqrt{48-32}}{2}=\frac{4\sqrt{3}\pm 4}{2}=2(\sqrt{3}\pm 1)$$

Thus, $BC=2(\sqrt{3}+1),\ BC'=2(\sqrt{3}-1)$

Difference of areas

$$=\frac{1}{2}\cdot 2(\sqrt{3}+1)\cdot 4\cdot\sin 30^\circ-\frac{1}{2}\cdot 2(\sqrt{3}-1)\cdot 4\cdot\sin 30^\circ$$

$$=\frac{1}{2}\cdot 4\times\frac{1}{2}\times 2[2]=4$$

24. (2) : $y = -2x + 1$ passes through nearest directrix

$$\Rightarrow\ 0=\frac{-2a}{e}+1\Rightarrow\frac{2a}{e}=1\Rightarrow\frac{a}{e}=\frac{1}{2}\ \Rightarrow e=2a$$

Now, $e^2=1+\dfrac{b^2}{a^2}\ \Rightarrow\ e^2=5-\dfrac{4}{e^2}\ \Rightarrow\ e^4-5e^2+4=0$

$\Rightarrow\ (e^2-1)(e^2-4)=0\quad \therefore\ e^2=4\ \Rightarrow\ e=2$

25. (0.77) : From the given table prime numbers are 2, 3, 5 and 7.

$\therefore\ P(E) = P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$

$= P(2) + P(3) + P(5) + P(7) = 0.62$

Now, $P(F) = P(1 \text{ or } 2 \text{ or } 3)$

$= P(1) + P(2) + P(3) = 0.50$

$\therefore\ P(E\cap F) = P(2 \text{ or } 3) = P(2) + P(3) = 0.35$

Now, $P(E\cup F) = P(E) + P(F) - P(E\cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

				9		1		7				8		2	6	3	9	7		
	4		7				6						1	3		2		8		
	1		5		6							7				8	1		5	
		3		8				9						1		9			4	
	5				7		4	2												1
6		9											4	5			8	2	7	
	6					7		1	5		3	6			3		5			
	3	1	4	5	9	2		6	1		7			4				1	2	
	9		6		1	5	3			2		1		7				5		
						8			7		2	4		6						
						3								2						
7	8			6	9	1		3	2				8	9	4		2	1	5	
		6	3				2	8											2	4
				8	5		6			1	8		4	5	8					
	2			7										3		4	9			
4		7	6					9						4	2				1	9
			1				3	5				6	9					3		
1			5										1			8	7	9		
2			8	9		3										6			7	
8					1	5	4	6						6	9				3	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

CBSE and NCERT to help classes IX, X students shortlist career options

The CBSE, in association with the NCERT, has come up with an aptitude test– 'Tamanna' (Try and Measure Aptitude and Natural Abilities) – for students of Classes IX and X for determining their area of interest.

However, the board has clarified that the aptitude test is suggestive and should not be used as the sole deciding factor for choosing subjects. The test will be carried out at the school level and a test booklet has been given to all schools, along with score keys.

This exercise is known as Know Your Aptitude (KYA). The preparing authority is the NCERT and the controlling authority is the CBSE. They also plan to train some teachers who can assess the test. The initiative is to make sure that Class IX and X students know their strength and weakness. The test will be self-analysis of sorts so that students choose the right subject.

Students, teachers and parents have been told that the aptitude test gives information related to the strength of students and there is no pass or fail criteria. The test should be taken voluntarily by the interested students and must not be used to impose any subject, courses of study and/or vocations on them.

This Aptitude Test covers seven dimensions. Those dimensions and their operational definitions are given in the adjoining table.

Seven Dimensions	Area of Interest
1. **Language Aptitude :** It is concerned with a person's ability to use and understand written language. It evaluates how well a student understands words and their synonyms, spells words correctly and identifies the correct meaning of the given proverbs/idioms.	Reading and writing such as teaching, journalism and media studies, advertising, law, library science, stenography, business development, etc.
2. **Abstract Reasoning :** It refers to a person's ability for logical and analytical thinking. This sub-test is non-verbal and it assesses how well students can reason and logically relate geometric shapes or designs.	Mathematics, computer programming, architecture, law, medicine, economics, mechanics forensic science, etc.
3. **Verbal Reasoning :** It is the ability to understand and reason using concepts expressed in words. It evaluates a student's ability to think constructively with words.	Psychology, speech therapist, auctioneering, advertising, linguistics, business, law, education, public relations, marketing, journalism, etc.
4. **Mechanical Reasoning :** It refers to a person's ability to understand and apply mechanical concepts principles to solve problems.	Machinery/ electrical/ civil automobile engineering, carpentry, electrician, machine operator, physics, chemistry, etc.
5. **Numerical Aptitude :** It refers to understanding numerical relationships and applying the same to the issue/problem. This sub-test assesses how well a student is able to solve problems covering four primary arithmetic operations like addition, subtraction, multiplication and division.	All types of engineering, architecture, oceanography, geology, meteorology, biosciences, health sciences and of course statistics and natural sciences.
6. **Spatial Aptitude :** It is related to the capacity to mentally manipulate actual materials through imagining. A student in this ability test is required to quickly judge how an object would look like when constructed in a given way.	Manufacturing industry, drafting, designing (fashion, interior, toys and games, jewellery, urban planning, landscape designing, etc.), architecture, astronomy, chemist, visual arts, animation, multimedia art, etc.
7. **Perceptual Aptitude :** It refers to a person's ability to quickly, accurately and meaningfully compare visual information, *i.e.,* letters, numbers, objects, pictures or patterns. In this sub-test, perceptual aptitude assesses how the students rapidly compare the paired groups of letters or numbers and identify the similarity or differences.	Bank-teller, accountants, computer programmers, police detectives, data entry, assembly work, record keeping, dispatching, filing, etc.

A cluster of aptitudes is required to perform effectively in a course of study or in an occupation. Therefore, the choice of a course of study or occupation should not be based on performance in one single aptitude only.

Low score in all subsets

To help such students, educational and career planning sessions maybe organised and they may also be referred to the school counsellor for career counselling.

According to the aptitude manual, several students may not score high on any of the seven subtests. This does not mean that they lack the ability to pursue further education or training in courses to choose a career. Such students need assistance for self-exploration and encouraging participation in various school activities of their interest, in addition to the subjects of study.

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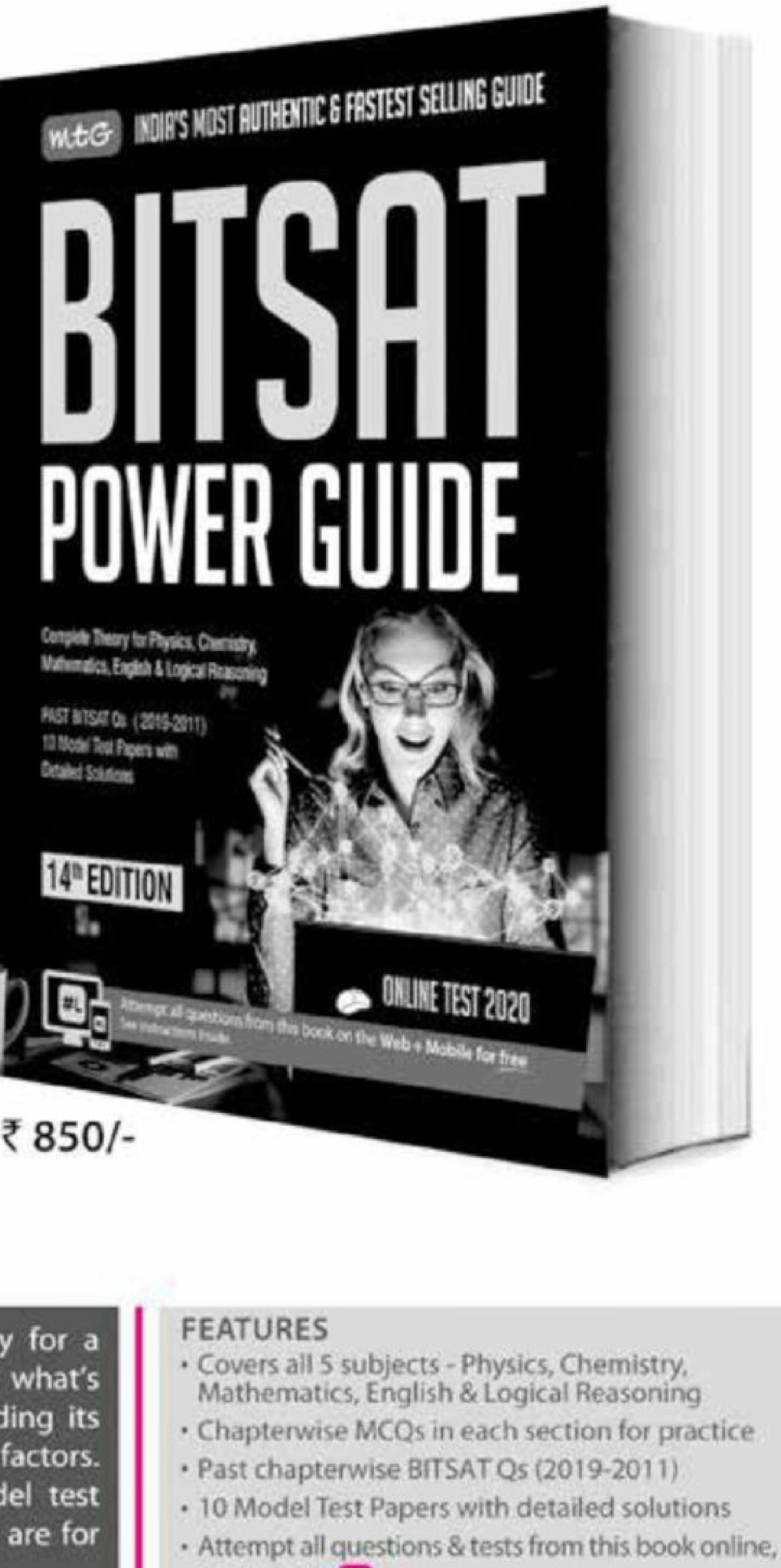
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CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 8

Three Dimensional Geometry

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All the questions are compulsory.

(ii) This question paper contains 36 questions.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION - A

Q.1-Q.10 are multiple choice type questions. Select the correct option.

1. The coordinates of a point P are (3, 12, 4) w.r.t. origin O, then the direction cosines of OP are

(a) 3, 12, 4 (b) $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

(c) $\frac{3}{\sqrt{13}}, \frac{1}{\sqrt{13}}, \frac{2}{\sqrt{13}}$ (d) $\frac{3}{13}, \frac{12}{13}, \frac{4}{13}$

2. The equation of a line passing through the point (–3, 2, –4) and equally inclined to the axes are

(a) $x - 3 = y + 2 = z - 4$

(b) $x + 3 = y - 2 = z + 4$

(c) $\frac{x+3}{1} = \frac{y-2}{2} = \frac{z+4}{3}$

(d) None of these

3. The angle between the planes $x + y = 0$ and $y - z = 1$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

4. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(a) 2 (b) 1 (c) –1 (d) –2

5. The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is

(a) 45° (b) 30° (c) 60° (d) 90°

6. An equation of the plane passing through the points (3, 2, –1), (3, 4, 2) and (7, 0, 6) is $5x + 3y - 2z = \lambda$, where λ is

(a) 23 (b) 21 (c) 19 (d) 27

7. The shortest distance between the lines $\frac{x}{m_1} = \frac{y}{1} = \frac{z-a}{0}$, $\frac{x}{m_2} = \frac{y}{1} = \frac{z+a}{0}$ is

(a) a (b) $\dfrac{a}{|m_1 - m_2|}$

(c) $2a$ (d) $\dfrac{2a}{|m_1 - m_2|}$

8. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

(a) 3 (b) 2 (c) 1 (d) 4

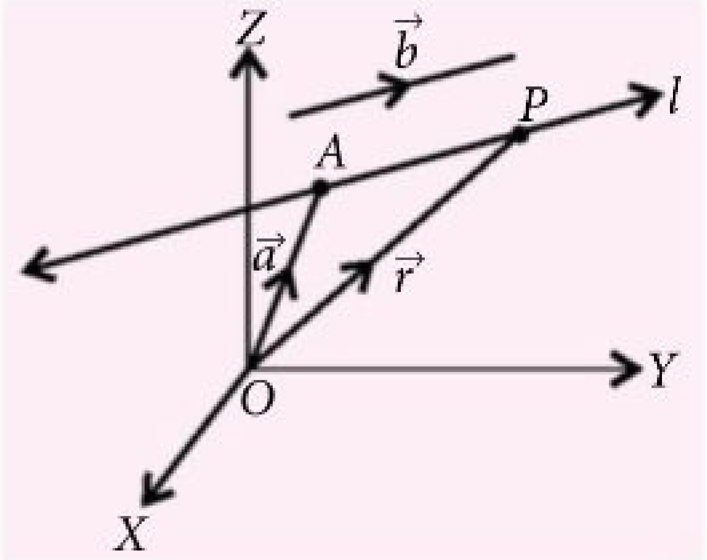
9. If lines $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$

and $\dfrac{x-1}{3k} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$

are mutually perpendicular, then k is equal to

(a) $-\dfrac{10}{7}$ (b) $-\dfrac{7}{10}$ (c) -10 (d) -7

10. In the figure, $\vec{a}$ is the position vector of the point A with respect to the origin O. l is a line parallel to a vector $\vec{b}$. The vector equation of line l is



(a) $\vec{r} = \lambda(\vec{a} \times \vec{b})$ (b) $\vec{r} = \lambda\vec{a} - \vec{b}$

(c) $\vec{r} = \vec{a} + \lambda\vec{b}$ (d) $\vec{r} = \lambda(\vec{a} \cdot \vec{b})$

(Q.11 - Q.15) Fill in the blanks.

11. The distance of the plane $2x - 3y + 6z + 14 = 0$ from the origin is ____________.

12. The cartesian equation of the plane $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ is ______ .

OR

If the angle between the planes $3x + 2y + z - 5 = 0$ and $x + y - 2z - 3 = 0$ is $\cos^{-1}\left(\dfrac{3}{2\sqrt{k}}\right)$, then $k =$ _______ .

13. A plane passes through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4), then the equation of plane is ______ .

14. The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are ______ .

15. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to _________.

OR

The equation of the line parallel to the line $\dfrac{x-2}{-3} = \dfrac{y+3}{2} = \dfrac{z+5}{6}$ and passing through the point (1, 2, 3) is _______ .

(Q.16-Q.20) Answer the following questions.

16. For what values of m, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $m\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

17. Write the vector equation of $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{6-z}{2}$.

18. Reduce the equation of the plane $3x + 5y - 4z = 60$ to intercept form and find its intercepts on the coordinate axes.

19. Show that the planes $2x + 6y + 6z = 7$ and $3x + 4y - 5z = 8$ are at right angles.

20. Find the distance of the plane $\vec{r} \cdot \left(\dfrac{2}{7}\hat{i} + \dfrac{3}{7}\hat{j} - \dfrac{6}{7}\hat{k}\right) = 1$ from the origin.

OR

Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis.

SECTION - B

21. Find the vector and cartesian equations of the plane which passes through the point (5, 2, –4) and perpendicular to the line with direction ratios 2, 3, –1.

OR

Find the angle between the two planes

$3x - 6y + 2z - 7 = 0$ and $2x + 2y - 2z - 5 = 0$.

22. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

23. Find a unit vector normal to the plane

$\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) + 14 = 0$.

24. Find the angle between the lines

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

OR

Find the value of λ for which the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear.

25. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

26. Find the cartesian equation of the plane passing through the points (1, 1, 2), (0, 2, 3), (4, 5, 6).

SECTION - C

27. Find the shortest distance between the lines L_1 and L_2 whose vector equations are

$$L_1 : \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$L_2 : \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

28. Find the image of the point (1, 6, 3) in the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}.$$

OR

Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

29. Find the direction ratios of the normal to the plane passing through the point (2, 1, 3) and the line of intersection of the planes $x + 2y + z = 3$ and $2x - y - z = 5$.

30. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ be equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, find the value of λ.

OR

Show that the lines $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ and $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ are coplanar.

31. The cartesian equations of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through $(2, -1, -1)$ which is parallel to the given line.

32. If the perpendicular distance of a plane from the origin is 1 and d.c'.s of normal vector to the plane satisfies $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then find the value of k.

SECTION - D

33. Find the vector and cartesian equations of a line passing through $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

34. Find the vector and cartesian equations of a plane containing the two lines

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k}) \text{ and}$$

$$\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k}).$$

Also show that the line $\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + p(3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane.

OR

Find the coordinate of the point P where the line through $A(3, -4, -5)$ and $B(2, -3, 1)$ crosses the plane passing through three points $L(2, 2, 1)$, $M(3, 0, 1)$ and $N(4, -1, 0)$. Also, find the ratio in which P divides the line segment AB.

35. Find the equations of two planes through the points (4, 2, 1), (2, 1, – 1) and making angles $\pi/4$ with the plane $x - 4y + z - 9 = 0$.

36. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ are coplanar.

Also find the equation of the plane containing them.

OR

Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$. Also find image of P in the plane.

SOLUTIONS

1. **(d) :** Direction ratios of OP are $(3 - 0, 12 - 0, 4 - 0)$ or $(3, 12, 4)$. Also $\sqrt{3^2 + 12^2 + 4^2} = 13$

$\therefore$ Direction cosines are $\left(\frac{3}{13}, \frac{12}{13}, \frac{4}{13}\right)$.

2. **(b) :** Since, line is equally inclined to the axes.

$\therefore \quad l = m = n$ ...(i)

The required equation of line passing through $(-3, 2, -4)$ is $\frac{x+3}{l} = \frac{y-2}{l} = \frac{z+4}{l}$ [Using (i)]

$$\Rightarrow \frac{x+3}{1} = \frac{y-2}{1} = \frac{z+4}{1} \Rightarrow x + 3 = y - 2 = z + 4$$

3. **(c) :** The d.r.'s of the two normals are <1, 1, 0> and <0, 1, –1>. The angle between the planes is

$$\theta = \cos^{-1}\left(\frac{1 \times 0 + 1 \times 1 - 0 \times 1}{\sqrt{2} \cdot \sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

4. **(a) :** Equation of line joining the points $(3, 2, -1)$ and $(6, 2, -2)$ is

$$\frac{x-3}{6-3} = \frac{y-2}{2-2} = \frac{z+1}{-2+1} \text{ i.e., } \frac{x-3}{3} = \frac{y-2}{0} = \frac{z+1}{-1} = \lambda \text{ (say)}$$

$\Rightarrow x = 3\lambda + 3, y = 2, z = -\lambda - 1$

So, y-coordinate of P is 2.

5. (d) : The direction ratios of lines are $< 2, 5, 4 >$ and $<1, 2, -3>$

$$\therefore \quad \cos\theta = \frac{2\cdot 1 + 5\cdot 2 + 4(-3)}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{(2+10-12)}{\sqrt{4+25+16}\sqrt{1+4+9}} = 0 \quad \Rightarrow \quad \theta = 90°$$

6. (a) : Equation of plane through (3, 2, –1), (3, 4, 2) and (7, 0, 6) is

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ 0 & 2 & 3 \\ 4 & -2 & 7 \end{vmatrix} = 0$$

i.e., $5x + 3y - 2z = 23 \ \therefore \ \lambda = 23$

7. (c) : Here, $\vec{a}_1 = a\hat{k}, \vec{b}_1 = m_1\hat{i} + \hat{j}$,

$\vec{a}_2 = -a\hat{k}, \ \vec{b}_2 = m_2\hat{i} + \hat{j}$

$\therefore \quad \vec{b}_1 \times \vec{b}_2 = (m_1 - m_2)\hat{k}$

$$\therefore \quad \text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = 2a$$

8. (b) : Here, the direction cosines of the given line are $\cos\alpha, \cos\beta, \cos\gamma$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow \quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

9. (a) : Lines $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$ and

$\dfrac{x-1}{3k} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ are perpendicular if

$-3(3k) + 2k + 2(-5) = 0 \ \Rightarrow \ k = -10/7$

10. (c) : The vector equation of line passing through point having position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is given by $\vec{\mu} = \vec{a} + \lambda\vec{b}$.

11. D.r.'s of the normal to the plane $2x - 3y + 6z + 14 = 0$ are $<2, -3, 6>$.

Now, $\sqrt{(2)^2 + (-3)^2 + (6)^2} = \sqrt{49} = 7$

$\therefore$ Required distance = 14/7 = 2

12. Given vector equation is, $\vec{r}\cdot(\hat{i} + \hat{j} - \hat{k}) = 2$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k})\cdot(\hat{i} + \hat{j} - \hat{k}) = 2 \ \Rightarrow \ x + y - z = 2$

which is the cartesian form of plane.

OR

The d.r.'s of two normals are $< 3, 2, 1 >$ and $< 1, 1, -2 >$

$\therefore$ Angle between the planes is

$$\cos^{-1}\left(\frac{3(1)+2(1)+1(-2)}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+1^2+(-2)^2}}\right) = \cos^{-1}\left(\frac{3}{2\sqrt{21}}\right)$$

$\therefore \quad k = 21$

13. The equation of plane passing through the points (2, 0, 0), (0, 3, 0) and (0, 0, 4) is $\dfrac{x}{2} + \dfrac{y}{3} + \dfrac{z}{4} = 1$.

14. Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are

$$\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}} \quad i.e., \quad \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$$

15. D.r.'s of *x*-axis is <1, 0, 0> and d.r.'s of the normal to the plane $2x - 3y + 6z = 11$ is <2, –3, 6>.

$$\text{Then, } \sin(\sin^{-1}\alpha) = \frac{2+0+0}{\sqrt{0^2+0^2+1^2}\sqrt{2^2+(-3)^2+6^2}}$$

$\Rightarrow \quad \alpha = 2/7$

OR

The equation of line parallel to line

$\dfrac{x-2}{-3} = \dfrac{y+3}{2} = \dfrac{z+5}{6}$ *i.e.*, $-3\hat{i} + 2\hat{j} + 3\hat{k}$ and passing through point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ is $\dfrac{x-1}{-3} = \dfrac{y-2}{2} = \dfrac{z-3}{6}$.

16. Given, $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = m\hat{i} + 6\hat{j} - 8\hat{k}$.

If $\vec{a}$ and $\vec{b}$ are collinear, then

$$\frac{m}{2} = \frac{6}{-3} = \frac{-8}{4} \Rightarrow \frac{m}{2} = -2 \Rightarrow m = -4$$

17. The line passes through point (5, –4, 6) and d.r.'s of the line are <3, 7, –2>.

$$\therefore \quad \text{Vector equation is } \vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$$

18. The equation of the given plane is

$$3x + 5y - 4z = 60 \Rightarrow \frac{x}{20} + \frac{y}{12} + \frac{z}{(-15)} = 1$$

So, the intercepts made by the plane with the coordinate axes are 20, 12 and –15 respectively.

19. We have, planes $2x + 6y + 6z = 7$ and $3x + 4y - 5z = 8$

Here, $a_1 = 2, b_1 = 6, c_1 = 6, a_2 = 3, b_2 = 4, c_2 = -5$

$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = (2)(3) + (6)(4) + (6)(-5) = 0$

So, planes are at right angles.

20. We have, $\vec{r}\cdot\left(\dfrac{2}{7}\hat{i} + \dfrac{3}{7}\hat{j} - \dfrac{6}{7}\hat{k}\right) = 1$

$$\text{Here, } \vec{n} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k} \quad \therefore \quad |\vec{n}| = \sqrt{\frac{49}{49}} = 1$$

So, $\vec{n}$ is a unit vector. Hence, given equation of plane is in normal form. So, distance from origin is 1.

OR

Any line parallel to *z*-axis has direction ratios proportional to <0, 0, 1>.

$\therefore$ The equation of a line through (α, β, γ) and parallel to z-axis is $\frac{x-\alpha}{0}=\frac{y-\beta}{0}=\frac{z-\gamma}{1}$.

21. The vector equation of the plane passing through the point whose position vector is $\vec{a}=(5\hat{i}+2\hat{j}-4\hat{k})$ and the normal vector $\vec{n}=(2\hat{i}+3\hat{j}-\hat{k})$ is given by $\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

$\Rightarrow\ \vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=(5\hat{i}+2\hat{j}-4\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k})=20$

Hence, the required vector equation of the plane is

$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=20$ ...(i)

Let $\vec{r}=(x\hat{i}+y\hat{j}+z\hat{k})$, then from (i), we have

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k})=20 \Rightarrow 2x+3y-z=20$

which is the required cartesian equation of the plane.

OR

Given equations of planes are $3x-6y+2z-7=0$ and $2x+2y-2z-5=0$

Now, the angle between two planes is

$$\cos^{-1}\left|\frac{3\times 2+(-6)\times 2+2\times(-2)}{\sqrt{3^2+(-6)^2+(2)^2}\sqrt{2^2+2^2+(-2)^2}}\right|$$

$$=\cos^{-1}\left|\frac{6-12-4}{\sqrt{9+36+4}\sqrt{4+4+4}}\right|=\cos^{-1}\left|\frac{-10}{\sqrt{49}\sqrt{12}}\right|$$

$$=\cos^{-1}\left(\frac{10}{7\times 2\sqrt{3}}\right)=\cos^{-1}\left(\frac{5\sqrt{3}}{21}\right)$$

22. Here $\vec{a}_1=\hat{i}+2\hat{j},\ \vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k},\ \vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

Now, $(\vec{a}_2-\vec{a}_1)=2\hat{i}+\hat{j}-5\hat{k}$

$\therefore\ \vec{b}\times(\vec{a}_2-\vec{a}_1)=-21\hat{i}+22\hat{j}-4\hat{k}$

$\therefore\ |\vec{b}\times(\vec{a}_2-\vec{a}_1)|=\sqrt{441+484+16}=\sqrt{941}$

and $|\vec{b}|=\sqrt{(2)^2+(3)^2+(6)^2}=\sqrt{49}=7$

$\therefore\ d=\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}=\frac{\sqrt{941}}{7}$ units.

23. We have, $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})+14=0$

$\Rightarrow\ \vec{r}\cdot(-2\hat{i}+3\hat{j}-6\hat{k})=14$

$\Rightarrow\ \vec{r}\cdot\vec{n}=14$, where $\vec{n}=(-2\hat{i}+3\hat{j}-6\hat{k})$

$\therefore\ |\vec{n}|=\sqrt{(-2)^2+3^2+(-6)^2}=7$

Hence, the unit vector normal to the given plane is

$$\hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{-2\hat{i}+3\hat{j}-6\hat{k}}{7}=\left(-\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)$$

24. Given lines are $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$

and $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$

On comparing with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ we get $\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$

$\therefore$ The angle between the lines is given by

$$\cos\theta=\left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|=\left|\frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}\right|$$

$$=\left|\frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}}\right|=\frac{19}{\sqrt{49}\sqrt{9}}=\frac{19}{21}$$

$\Rightarrow\ \theta=\cos^{-1}\left(\frac{19}{21}\right)$

OR

The equation of the line AB is

$$\frac{x+1}{-4+1}=\frac{y-3}{2-3}=\frac{z-2}{-2-2}\Rightarrow\frac{x+1}{-3}=\frac{y-3}{-1}=\frac{z-2}{-4} \quad ...(i)$$

Since the points A, B and C are collinear, so the point $C(5, 5, \lambda)$ lies on (i).

$\therefore\ \frac{5+1}{-3}=\frac{5-3}{-1}=\frac{\lambda-2}{-4}\ \Rightarrow\ \frac{\lambda-2}{-4}=-2$

$\Rightarrow\ \lambda-2=8\ \Rightarrow\ \lambda=10$

25. Vector equation of the line passing through $(1, 2, -1)$ and parallel to the line $5x-25=14-7y=35z$

i.e., $\frac{x-5}{1/5}=\frac{y-2}{-1/7}=\frac{z}{1/35}$ or $\frac{x-5}{7}=\frac{y-2}{-5}=\frac{z}{1}$

is $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$.

26. Let $A(x_1, y_1, z_1)\equiv(1, 1, 2)$, $B(x_2, y_2, z_2)\equiv(0, 2, 3)$, $C(x_3, y_3, z_3)\equiv(4, 5, 6)$

Equation of plane is given by

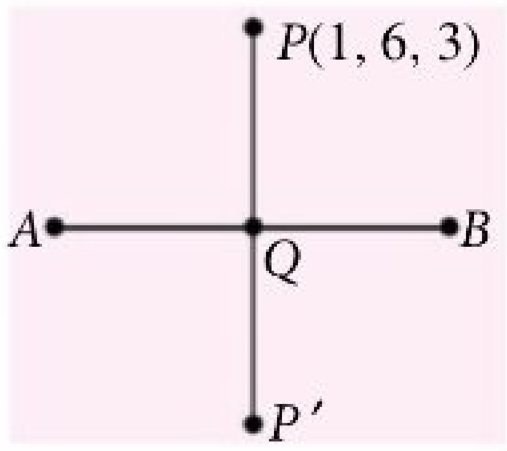
$$\begin{vmatrix} x-1 & y-1 & z-2 \\ -1 & 1 & 1 \\ 3 & 4 & 4 \end{vmatrix}=0$$

$\Rightarrow\ (x-1)(4-4)-(y-1)(-4-3)+(z-2)(-4-3)=0$

$\Rightarrow\ y-1-z+2=0\ \Rightarrow\ y-z+1=0$

27. *Refer to answer 53, Page no. 283 of MTG CBSE Champion Mathematics, Class-12*

28. Let the given point be $P(1, 6, 3)$ and the given line be AB whose equation is $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$



Draw $PQ \perp AB$ and produce it to P' such that $QP'=PQ$. Then P' is the image of P in the line AB. Coordinates of Q are $(r, 2r+1, 3r+2)$ for some value of r.

D.r.'s of PQ are $<r-1, 2r-5, 3r-1>$

D.r.'s of AB are $<1, 2, 3>$.

Since $PQ \perp AB$ $\therefore$ $1(r-1)+2(2r-5)+3(3r-1)=0$

$\Rightarrow$ $14r-14=0 \Rightarrow r=1$

$\therefore$ Coordinates of Q are $(1, 3, 5)$.

Let the coordinates of P' be (α, β, γ).

Since Q is the mid-point of PP'

$$\therefore \quad \frac{\alpha+1}{2}=1, \frac{\beta+6}{2}=3, \frac{\gamma+3}{2}=5 \Rightarrow \alpha=1, \beta=0, \gamma=7$$

Hence, the coordinates of P' are $(1, 0, 7)$

OR

Refer to answer 39, Page no. 281 of MTG CBSE Champion Mathematics, Class-12

29. The equation of the plane passing through the line of intersection of the planes $x+2y+z=3$ and $2x-y-z=5$ is given by

$$(x+2y+z-3)+\lambda(2x-y-z-5)=0$$

$$\Rightarrow x(2\lambda+1)+y(2-\lambda)+z(1-\lambda)-3-5\lambda=0 \quad \text{...(i)}$$

It passes through $(2, 1, 3)$, then

$$2(2\lambda+1)+(2-\lambda)+3(1-\lambda)-3-5\lambda=0$$

$\Rightarrow$ $4-5\lambda=0 \Rightarrow \lambda=4/5$

Substituting $\lambda=4/5$ in (i), we get $13x+6y+z-35=0$ as the equation of the required plane. Direction ratios of normal to this plane are proportional to 13, 6, 1.

30. It is given that the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$.

$$\therefore \quad \left|\frac{(\hat{i}+\hat{j}+\lambda\hat{k})\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13}{\sqrt{9+16+144}}\right|$$

$$=\left|\frac{(-3\hat{i}+0\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13}{\sqrt{9+16+144}}\right|$$

$$\Rightarrow \quad \left|\frac{3+4-12\lambda+13}{13}\right|=\left|\frac{-9+0-12+13}{13}\right|$$

$\Rightarrow$ $|20-12\lambda|=8 \Rightarrow 20-12\lambda=\pm 8$

$\Rightarrow$ $12\lambda=12$ or $12\lambda=28 \Rightarrow \lambda=1$ or $\lambda=7/3$

OR

The given lines are

$$\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7} \text{ and } \frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$$

Here, $x_1=2, y_1=4, z_1=6, x_2=-1, y_2=-3, z_2=-5$, $a_1=1, b_1=4, c_1=7, a_2=3, b_2=5, c_2=7$

$$\text{Now, } \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} -3 & -7 & -11 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$=(-3)(28-35)+7(7-21)-11(5-12)$

$=21-98+77=0$

$\therefore$ Given lines are coplanar.

31. *Refer to answer 45, Page no. 282 of MTG CBSE Champion Mathematics, Class-12*

32. Let d.c'.s of the normal to the plane be $< l, m, n >$.

Then equation of plane is $lx+my+nz=1$ ...(i)

Also, $l^2+m^2+n^2=1$ ...(ii)

It is given that $< l, m, n >$ satisfy the equation

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k \Rightarrow \frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}=k$$

$$\Rightarrow \quad (l^2+m^2+n^2)\left\{\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}\right\}=k$$

$$\text{Now, } \frac{l^2+m^2+n^2}{3} \ge \frac{3}{\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}} \quad (\because \text{A.M.} \ge \text{H.M.})$$

$$\Rightarrow \quad (l^2+m^2+n^2)\left(\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}\right) \ge 9 \Rightarrow k \ge 9.$$

33. Let the equation of line passing through $(1, 2, -4)$ and perpendicular to the lines

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

$$\text{be } \frac{x-1}{l}=\frac{y-2}{m}=\frac{z+4}{n} \quad \text{...(i)}$$

$\therefore$ $l(3)+m(-16)+n(7)=0$ and $l(3)+m(8)+n(-5)=0$

$$\Rightarrow \quad \frac{l}{80-56}=\frac{m}{21+15}=\frac{n}{24+48} \Rightarrow \frac{l}{2}=\frac{m}{3}=\frac{n}{6}$$

$\therefore$ The equation of the required line is

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}.$$

and its vector equation is $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$.

34. The given lines are

$$\vec{r}=2\hat{i}+\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+5\hat{k}) \quad \text{...(i)}$$

$$\text{and } \vec{r}=3\hat{i}+3\hat{j}+2\hat{k}+\mu(3\hat{i}-2\hat{j}+5\hat{k}) \quad \text{...(ii)}$$

Here, $\vec{a}_1=2\hat{i}+\hat{j}-3\hat{k}, \vec{b}_1=\hat{i}+2\hat{j}+5\hat{k}$

$\vec{a}_2=3\hat{i}+3\hat{j}+2\hat{k}, \vec{b}_2=3\hat{i}-2\hat{j}+5\hat{k}$

The plane containing lines (i) and (ii) will pass through $\vec{a}_1=2\hat{i}+\hat{j}-3\hat{k}$.

Also the plane is parallel to two vectors $\vec{b}_1$ and $\vec{b}_2$.

$\therefore$ The plane is normal to the vector

$$\vec{n}=\vec{b}_1\times\vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}=20\hat{i}+10\hat{j}-8\hat{k}$$

$\therefore$ The vector equation of the required plane is

$(\vec{r}-\vec{a}_1)\cdot\vec{n}=0 \Rightarrow \vec{r}\cdot\vec{n}=\vec{a}_1\cdot\vec{n}$

$$\Rightarrow \quad \vec{r}\cdot(20\hat{i}+10\hat{j}-8\hat{k})=(2\hat{i}+\hat{j}-3\hat{k})\cdot(20\hat{i}+10\hat{j}-8\hat{k})$$

$$\Rightarrow \quad \vec{r}\cdot(10\hat{i}+5\hat{j}-4\hat{k})=37 \quad \text{...(iii)}$$

Its cartesian equation is $10x + 5y - 4z = 37$.

The given line is $\vec{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + p(3\hat{i} - 2\hat{j} + 5\hat{k})$ ...(iv)

The line (iv) will lie in the plane (iii) if the plane passes through the point $\vec{a} = 2\hat{i} + 5\hat{j} + 2\hat{k}$ on line (iv) and is parallel to the line (iv).

Now, $\vec{a} \cdot (10\hat{i} + 5\hat{j} - 4\hat{k})$

$= (2\hat{i} + 5\hat{j} + 2\hat{k}) \cdot (10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$

$\therefore$ $\vec{a}$ lies in the plane (iii)

Also, $(10\hat{i} + 5\hat{j} - 4\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 5\hat{k})$

$= 10(3) + 5(-2) - 4(5) = 0$

$\therefore$ Line (iv) is parallel to the plane (iii).

$\therefore$ Line (iv) lies in the plane (iii).

OR

Refer to answer 71, Page no. 287 of MTG CBSE Champion Mathematics, Class-12

35. Any plane through the point (4, 2, 1) is

$A(x - 4) + B(y - 2) + C(z - 1) = 0$ ...(i)

Since it passes through the point (2, 1, – 1), we have

$A(2 - 4) + B(1 - 2) + C(-1 - 1) = 0$

or $2A + B + 2C = 0$ ...(ii)

As plane (i) makes an angle of $\pi/4$ with the plane $x - 4y + z - 9 = 0$, we have

$$\frac{|1A + (-4)B + 1C|}{\sqrt{1+16+1}\sqrt{A^2 + B^2 + C^2}} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$\Rightarrow |A - 4B + C| = 3\sqrt{A^2 + B^2 + C^2}$

$\Rightarrow 9(A^2 + B^2 + C^2) = (A - 4B + C)^2$ ...(iii)

Putting $B = -2A - 2C$ from (ii) in (iii), we get

$9(A^2 + 4A^2 + 4C^2 + 8AC + C^2) = (9A + 9C)^2$

$\Rightarrow 2A^2 + 5AC + 2C^2 = 0 \Rightarrow (2A + C)(A + 2C) = 0$

$\Rightarrow A = -C/2$ or $-2C$

From (ii), when $A = -C/2$, $B = -C$ and when $A = -2C$, $B = 2C$.

Substituting these values in (i), we get

$$-\frac{C}{2}(x - 4) - C(y - 2) + C(z - 1) = 0$$

$\Rightarrow x + 2y - 2z - 6 = 0$

and $-2C(x - 4) + 2C(y - 2) + C(z - 1) = 0$ $(C \neq 0)$

$\Rightarrow 2x - 2y - z - 3 = 0$

36. The given lines are

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

Here, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$, $\vec{b}_1 = 3\hat{i} - \hat{j}$

$\vec{a}_2 = 4\hat{i} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + 3\hat{k}$

We know that the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if $[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2] = 0$

$\Rightarrow [\vec{a}_1\ \vec{b}_1\ \vec{b}_2] = [\vec{a}_2\ \vec{b}_1\ \vec{b}_2]$.

$$\text{Now, } [\vec{a}_1\ \vec{b}_1\ \vec{b}_2] = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 1(-3) - 1(9) - 1(2) = -14$$

$$\text{and } [\vec{a}_2\ \vec{b}_1\ \vec{b}_2] = \begin{vmatrix} 4 & 0 & -1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 4(-3) - 1(2) = -14$$

Hence, $[\vec{a}_1\ \vec{b}_1\ \vec{b}_2] = [\vec{a}_2\ \vec{b}_1\ \vec{b}_2]$

$\Rightarrow$ The given lines are coplanar.

Now the eq. of the plane containing the given lines is

$(\vec{r} - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$ or $[\vec{r}\ \vec{b}_1\ \vec{b}_2] = [\vec{a}_1\ \vec{b}_1\ \vec{b}_2]$

$\Rightarrow \vec{r} \cdot (\vec{b}_1 \times \vec{b}_2) = -14$ ...(i)

$$\text{Since, } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = -3\hat{i} - 9\hat{j} + 2\hat{k}$$

From (i), $\vec{r} \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k}) = -14$

$\Rightarrow \vec{r} \cdot (3\hat{i} + 9\hat{j} - 2\hat{k}) = 14$.

OR

Refer to answer 102, Page no. 293 of MTG CBSE Champion Mathematics, Class-12

SAMURAI SUDOKU

ANSWER · NOVEMBER 2019

2	8	6	7	3	5	4	9	1				3	2	5	1	7	9	4	8	6
4	5	9	2	6	1	7	3	8				8	1	9	3	6	4	2	7	5
1	7	3	8	9	4	2	6	5				6	7	4	5	2	8	9	1	3
6	1	7	5	2	3	9	8	4				7	6	1	2	3	5	8	4	9
5	4	2	1	8	9	6	7	3				4	3	2	8	9	7	6	5	1
9	3	8	4	7	6	1	5	2				9	5	8	4	1	6	3	2	7
7	6	4	3	1	8	5	2	9	6	4	7	1	8	3	9	5	2	7	6	4
8	2	5	9	4	7	3	1	6	8	9	2	5	4	7	6	8	3	1	9	2
3	9	1	6	5	2	8	4	7	5	1	3	2	9	6	7	4	1	5	3	8
						6	8	5	3	2	4	9	7	1						
						1	3	2	7	8	9	6	5	4						
						7	9	4	1	5	6	3	2	8						
7	6	8	1	2	9	4	5	3	9	7	1	8	6	2	9	1	7	3	4	5
2	4	3	5	8	6	9	7	1	2	6	8	4	3	5	8	2	6	7	1	9
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9	3	7	4	6	1	5	8	2				1	2	7	4	3	9	6	5	8
3	5	6	7	1	2	8	9	4				9	5	4	2	6	8	1	7	3
8	7	1	6	9	4	3	2	5				3	7	1	5	9	4	2	8	6
4	9	2	8	5	3	6	1	7				2	8	6	1	7	3	5	9	4

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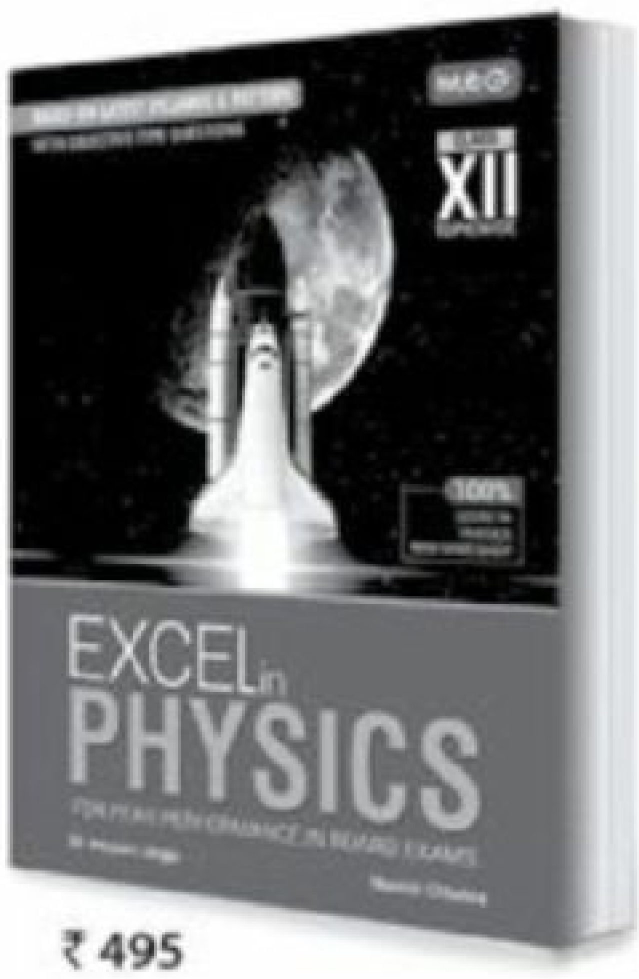
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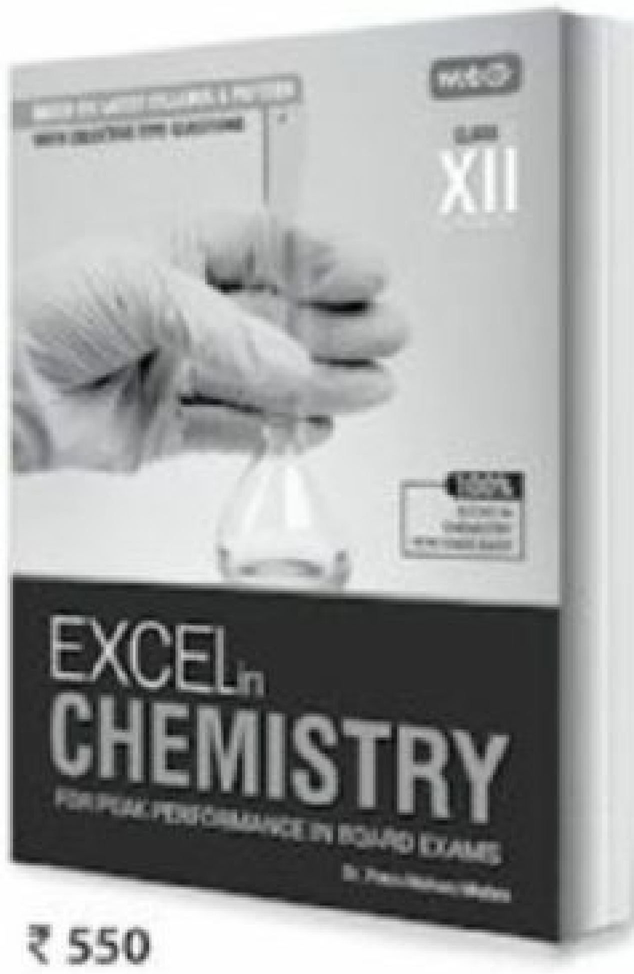
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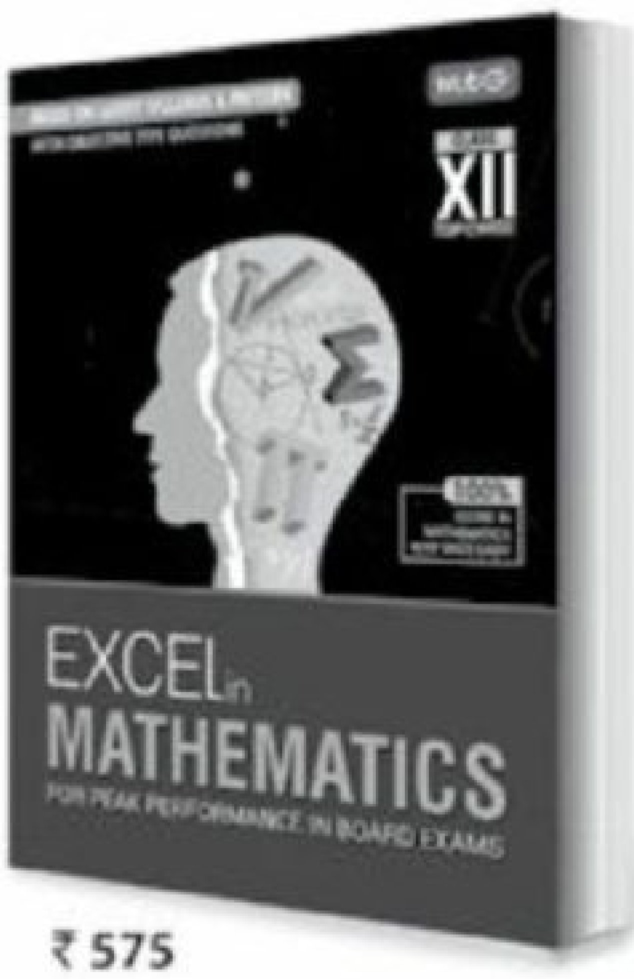
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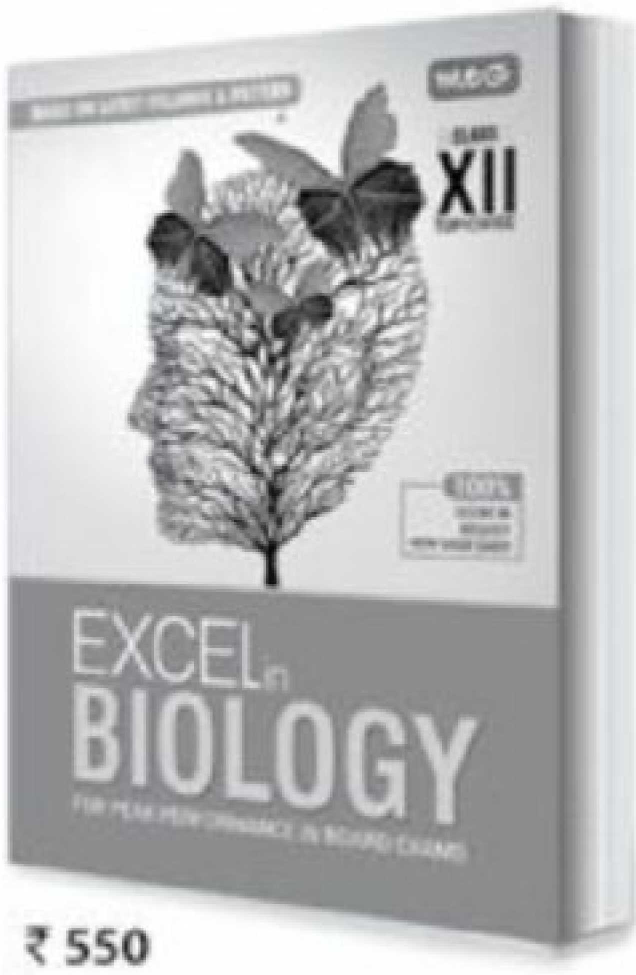
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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 7 : Definite Integration, Application of Integrals

Time Taken : 60 Min.

Only One Option Correct Type

1. If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then $I_8 + I_6$ equals

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

2. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is

(a) $\pi \log 2$ (b) $\frac{\pi}{8\log 2}$

(c) $\frac{\pi}{2\log 2}$ (d) $\log 2$

3. The value of $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 3 (d) 5

4. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$ is equal to

(a) 2 (b) 4 (c) 1 (d) 6

5. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

6. Let C be a curve passing through $M(2, 2)$ such that the slope of the tangent at any point to the curve is reciprocal of the ordinate of the point. If the area bounded by curve C and line $x = 2$ is A sq. unit, then the value of $\frac{3A}{2}$ is

(a) 8 (b) 9

(c) 10 (d) 11

One or More Than One Option(s) Correct Type

7. The value of $\int_{1/e}^{\tan x} \frac{t}{1+t^2} dt + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$ is

(a) $\frac{1}{2+\tan^2 x}$ (b) 1

(c) $\frac{\pi}{4}$ (d) $\frac{2}{\pi}\int_{-1}^{1} \frac{dt}{1+t^2}$

8. The area enclosed between the curves, $x^2 = y$ and $y^2 = x$ is equal to

(a) $\frac{1}{3}$ sq. unit (b) $2\int_0^1 (x-x^2)\,dx$

(c) area of the region $\{(x, y) : x^2 \le y \le |x|\}$

(d) None of these

9. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is (are)

(a) $\int_0^{\pi/4} x\, f(x)\, dx = \frac{1}{12}$ (b) $\int_0^{\pi/4} f(x)\, dx = 0$

(c) $\int_0^{\pi/4} x\, f(x)\, dx = \frac{1}{6}$ (d) $\int_0^{\pi/4} f(x)\, dx = 1$

10. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}dx$ is

(a) $\frac{\pi}{4}+2\log 2-\tan^{-1}2$ (b) $\frac{\pi}{4}+2\log 2-\tan^{-1}\frac{1}{3}$

(c) $2\log 2-\cot^{-1}3$

(d) $-\frac{\pi}{4}+\log 4+\cot^{-1}2$

11. Let $\int_\alpha^\beta \frac{f(\alpha+\beta-x)}{f(x)+f(\alpha+\beta-x)}dx=4$, then

(a) $\alpha=-1, \beta=7$ (b) $\alpha=5, \beta=13$

(c) $\alpha=-2, \beta=6$ (d) $\alpha=-10, \beta=-2$

12. For which of the following values of m, is the area of the region bounded by the curve $y=x-x^2$ and the line $y=mx$ equals 9/2?

(a) -4 (b) -2 (c) 2 (d) 4

13. The parabola $y^2=4x$ and $x^2=4y$ divide the square region bounded by the lines $x=4$, $y=4$ and the coordinate axes. If S_1, S_2, S_3 are the areas of these parts numbered from top to bottom respectively, then

(a) $S_1:S_2=1:1$ (b) $S_2:S_3=1:2$

(c) $S_1:S_3=1:1$ (d) $S_2:S_3=2:1$

Comprehension Type

Let $f(x)$ be a continuous function defined on the closed interval $[a, b]$, then

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right)=\int_0^1 f(x)\,dx$$

14. The value of $\lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{n+1}+\frac{2}{n+2}+\ldots+\frac{3n}{4n}\right\}$ is

(a) $5-2\ln 2$ (b) $4-2\ln 2$

(c) $3-2\ln 2$ (d) $2-2\ln 2$

15. The value of $\lim_{n\to\infty}\left\{\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots+\frac{1}{8n}\right\}$ is

(a) $\frac{5}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$

Matrix Match Type

16. Match the columns :

Column-I		Column-II	
P.	The area bounded by the curve $y=x\lvert x\rvert$, x-axis and the ordinates $x=1$, $x=-1$ is	1.	$\frac{10}{3}$ sq. units
Q.	The area of the region lying between the lines $x-y+2=0$, $x=0$ and the curve $x=\sqrt{y}$ is	2.	2 sq. units
R.	The area enclosed between the curves $y^2=x$ and $y=\lvert x\rvert$ is	3.	$\frac{2}{3}$ sq. unit
S.	The area bounded by the curves $y=\lvert x\rvert-1$ and $y=-\lvert x\rvert+1$ is	4.	$\frac{1}{6}$ sq. unit

	P	Q	R	S
(a)	1	3	4	2
(b)	3	1	4	2
(c)	2	1	3	4
(d)	4	1	3	2

Numerical Value Type

17. If $f(x)=\int_a^x \frac{1}{f(x)}dx$ and $\int_a^1 \frac{1}{f(x)}dx=\sqrt{2}$, then the value of $f(200)$ must be

18. The area enclosed by the curve $xy^2=4(2-x)$ and y-axis is λ sq. unit, then the value of $5050\cos(20\,\lambda)$ must be

19. The value of $\dfrac{2^{2010}\int_0^1 x^{1004}(1-x)^{1004}dx}{1005\int_0^1 x^{1004}(1-x^{2010})^{1004}dx}$ is

20. If the area of the region $\{(x, y) : 0 \le y \le x^2+1, 0 \le y \le x+1, 0 \le x \le 2\}$ is A, then the value of $6A-17$ is

◆ ◆

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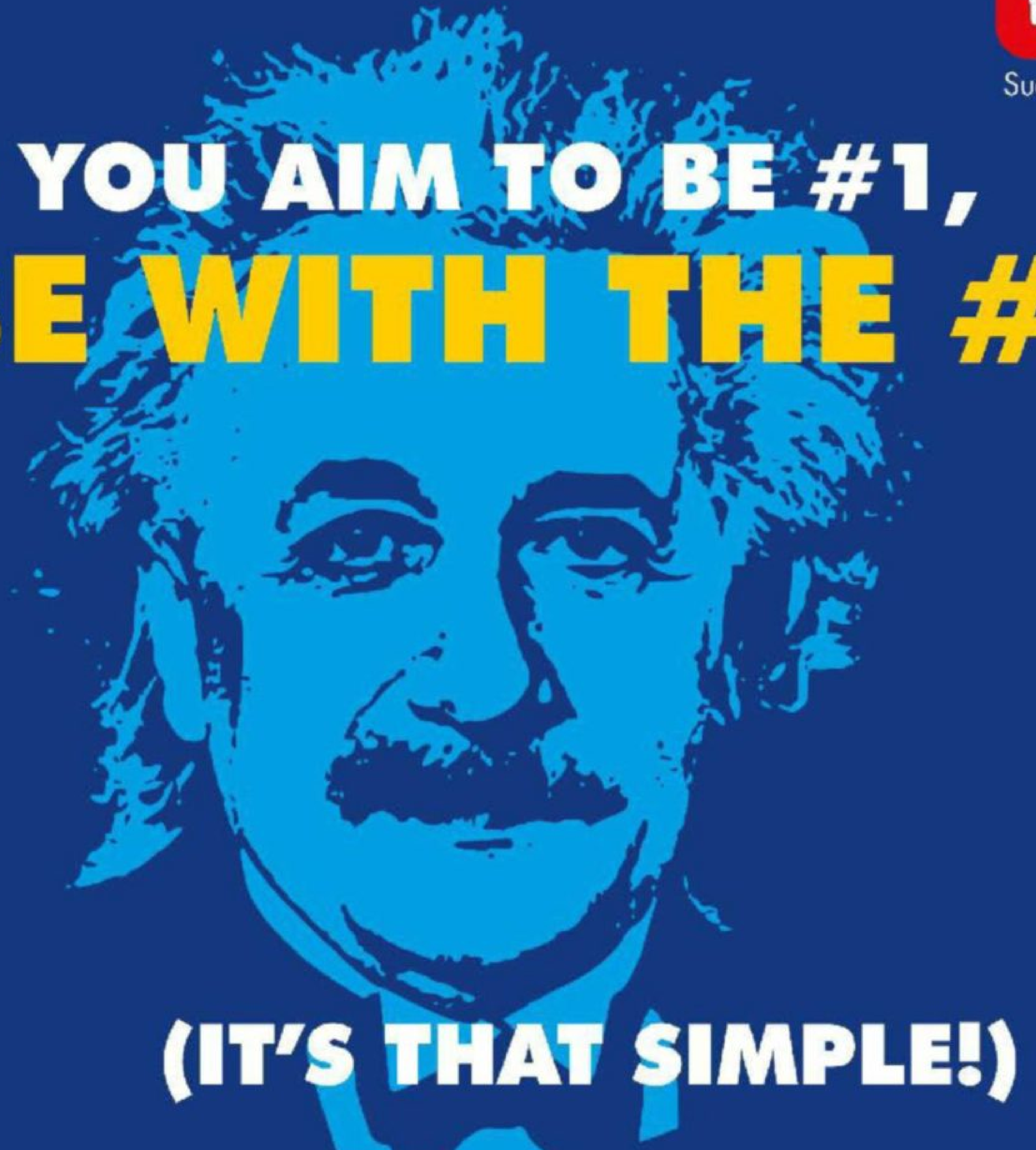

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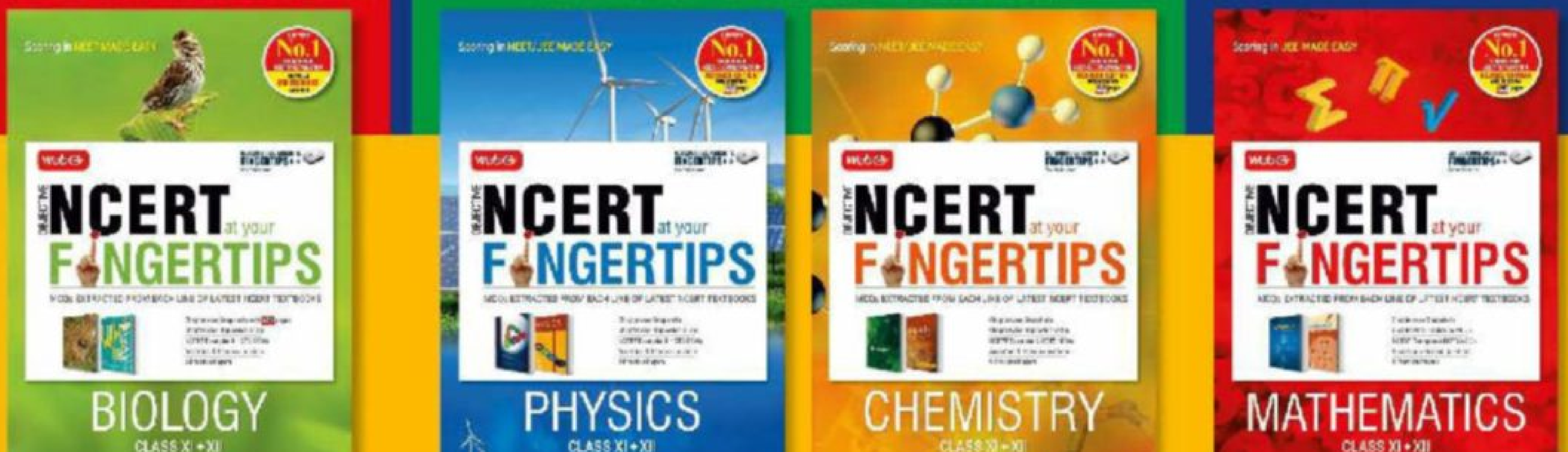
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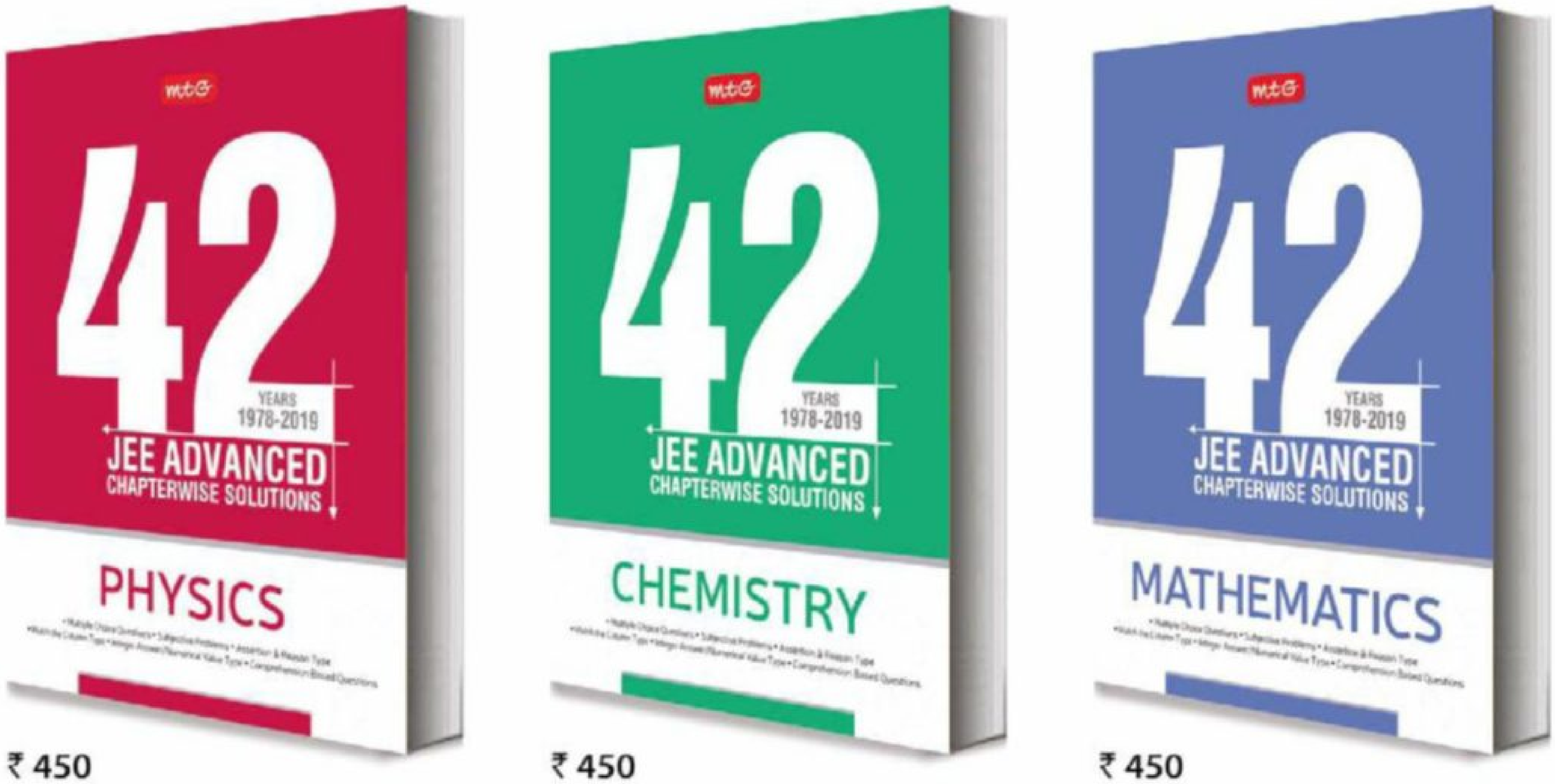

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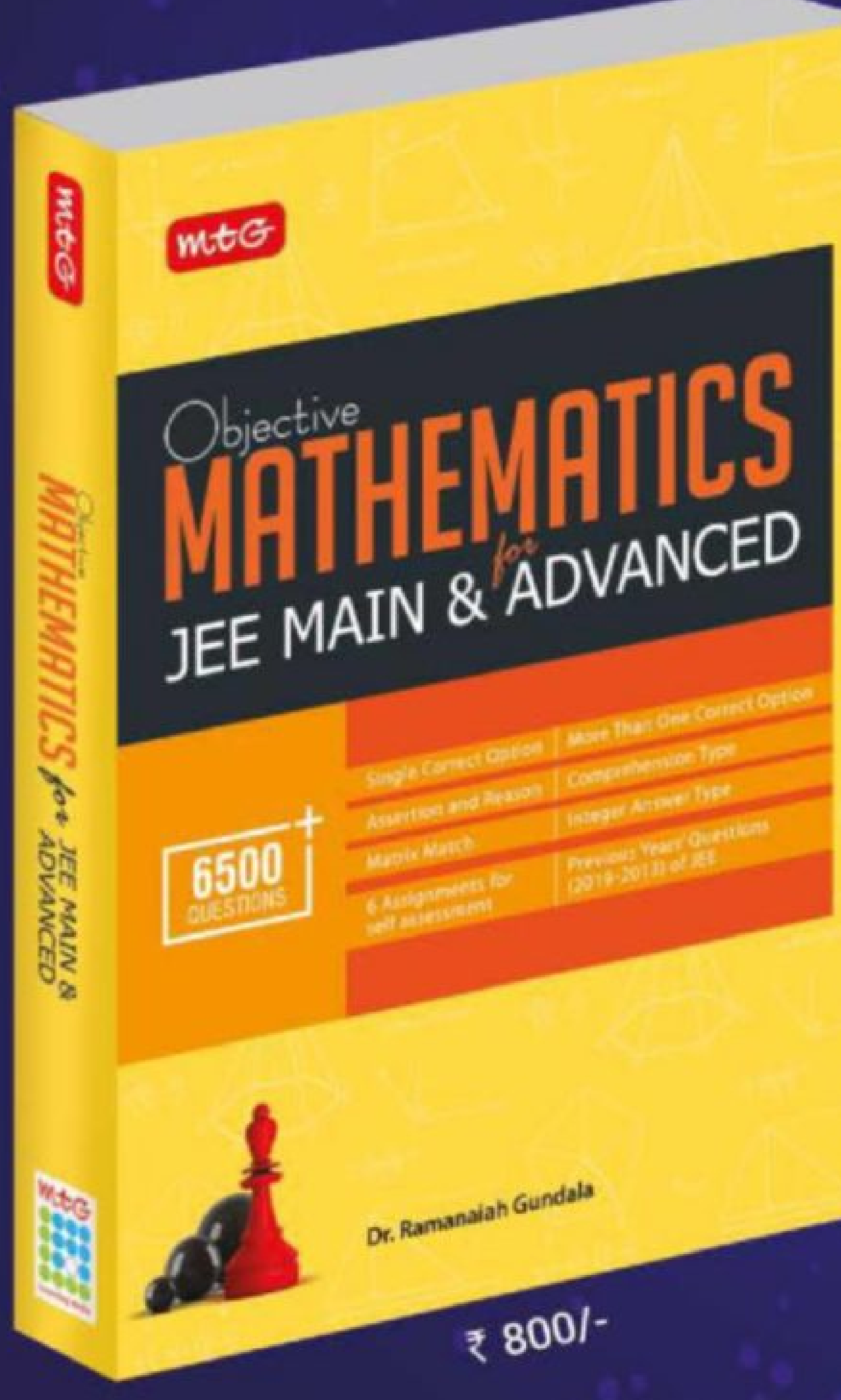
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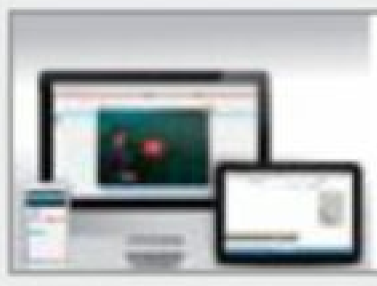

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